

Volume of Cylinders, Cones and Spheres: math lesson, Grade 8 – download the PDF

How much water fits in a soup can, an ice cream cone, or a basketball? Each question asks for a **volume**: the amount of space inside a solid. In this chapter you will learn three powerful formulas, see how they connect to each other, and use them to solve real-world problems. You already know that a rectangular prism has volume $V = Bh$. Now you will extend that idea to round solids.

1. Volume and Cubic Units

Volume counts how many unit cubes it takes to fill a solid. A unit cube with edges of 1 centimeter has a volume of 1 cubic centimeter, written 1 cm^3 . In the U.S. system we use cubic inches (in^3), cubic feet (ft^3), and gallons. In the metric system we use cm^3 , m^3 , milliliters (mL), and liters (L).

D

Volume of a prism or cylinder

When a solid has two identical parallel bases and the same cross-section all the way up, its volume is

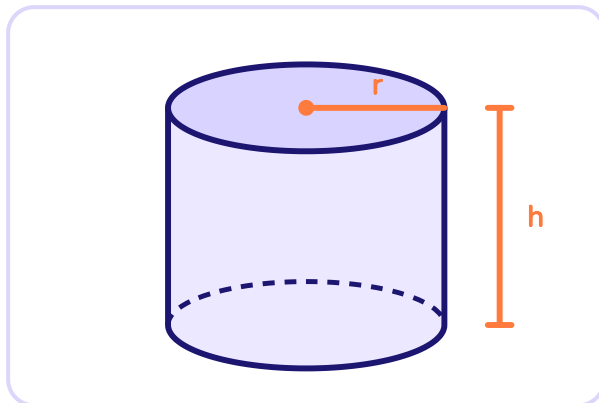
$$V = B \times h$$

where B is the area of one base and h is the perpendicular height.

Useful conversions: $1 \text{ cm}^3 = 1 \text{ mL}$, $1 \text{ L} = 1,000 \text{ cm}^3$, $1 \text{ m}^3 = 1,000 \text{ L}$, and $1 \text{ ft}^3 \approx 7.48$ gallons.

2. Volume of a Cylinder

A cylinder has two parallel circular bases. Because the base is a circle of radius r , its area is $B = \pi r^2$. Stacking that circle up to the height h gives the formula below.



P Volume of a cylinder

$$V = \pi r^2 h$$

where r is the radius of the base and h is the height. The height must be measured at a right angle to the bases.

E Example 1: a cylinder

A can has radius **3 cm** and height **10 cm**. Find its volume.

$V = \pi \times 3^2 \times 10 = 90\pi \text{ cm}^3$. With $\pi \approx 3.14$, $V \approx 3.14 \times 90 = 282.6 \text{ cm}^3$. The can holds about **282.6 mL**.

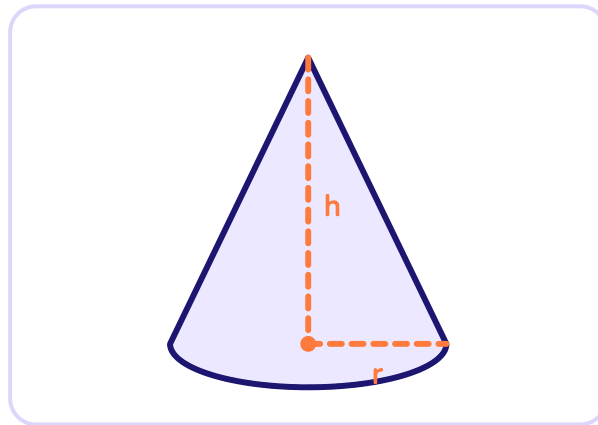


Watch out: radius versus diameter

The formula needs the *radius*. If a problem gives the diameter, cut it in half first. A cylinder with diameter 10 m has radius 5 m, not 10 m.

3. Volume of a Cone

A cone has one circular base and a single point called the apex. The height ***h*** is the perpendicular distance from the apex straight down to the base, not the slanted side.



Volume of a cone

$$V = \frac{1}{3}\pi r^2 h$$

where ***r*** is the radius of the base and ***h*** is the perpendicular height.

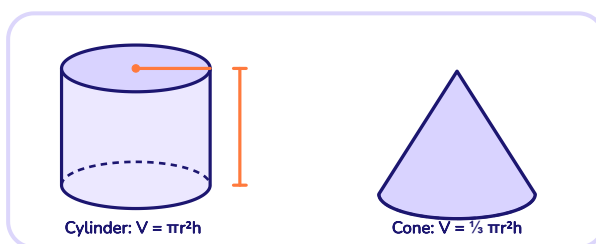
E Example 2: a cone

A paper party hat is a cone with radius **4 in** and height **9 in**.

$$V = \frac{1}{3}\pi \times 4^2 \times 9 = \frac{1}{3}\pi \times 144 = 48\pi \text{ in}^3 \approx 150.7 \text{ in}^3.$$

4. How Cones and Cylinders Are Related

Imagine a cone and a cylinder with exactly the same base radius and the same height. If you fill the cone with sand and pour it into the cylinder, you must do it three times to fill the cylinder to the top. That is why the cone formula has the extra factor $\frac{1}{3}$.



P Cone and cylinder with the same base and height

$$V_{\text{cone}} = \frac{1}{3} V_{\text{cylinder}} \quad \text{and} \quad V_{\text{cylinder}} = 3 V_{\text{cone}}$$

Solid ($r = 5 \text{ cm}$, $h = 12 \text{ cm}$)	Volume formula	Exact volume	Approximate ($\pi \approx 3.14$)
Cylinder	$\pi r^2 h$	$300\pi \text{ cm}^3$	942 cm^3
Cone	$\frac{1}{3}\pi r^2 h$	$100\pi \text{ cm}^3$	314 cm^3

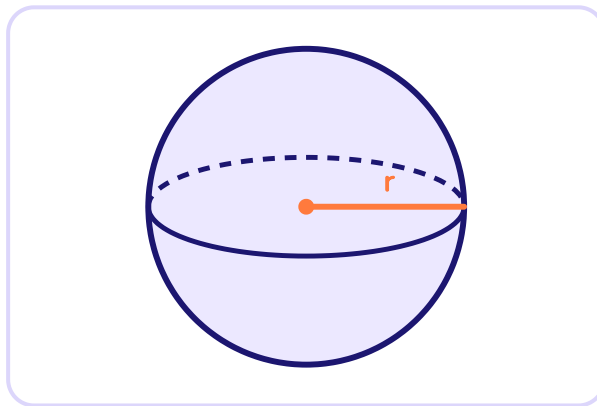


Zyro's tip

On my planet we say: "Three cones make one cylinder." If you already know one of the two volumes, you can find the other by multiplying or dividing by 3 instead of starting over.

5. Volume of a Sphere

A sphere is the set of all points in space at the same distance r from a center point. A ball is a good model. Notice that a sphere has no height and no flat base, so only the radius matters.



Volume of a sphere

$$V = \frac{4}{3}\pi r^3$$

A **hemisphere** is half of a sphere, so its volume is $\frac{2}{3}\pi r^3$.

E**Example 3: a sphere**

A decorative globe has radius **6 ft**. Then

$$V = \frac{4}{3}\pi \times 6^3 = \frac{4}{3}\pi \times 216 = 288\pi \text{ ft}^3 \approx 904.3 \text{ ft}^3.$$

**Watch out: cube, not square**

The radius is *cubed* ($r^3 = r \times r \times r$), not squared. Doubling the radius of a sphere makes the volume $2^3 = 8$ times larger.

6. Solving for an Unknown Dimension

Often the volume is given and you must find a radius or a height. Write the formula, substitute what you know, and solve the equation step by step.

M**Method: find a missing dimension**

1. Write the correct volume formula.
2. Substitute the known values (convert diameter to radius if needed).
3. Isolate the unknown: divide by the numbers and by π .
4. For r^2 take a square root; for r^3 take a cube root.
5. Write the answer with its unit and check it in the formula.

E Example 4: unknown radius of a cylinder

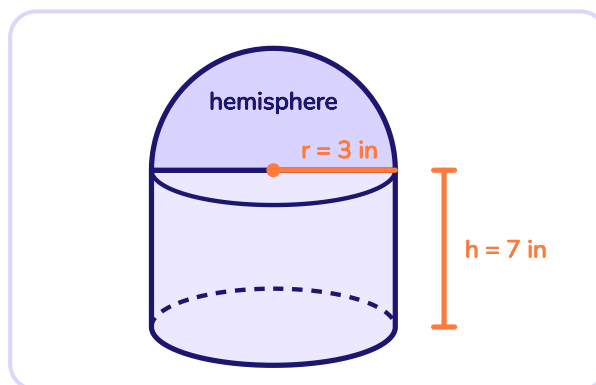
A cylinder has volume $200\pi \text{ cm}^3$ and height 8 cm . Then $\pi r^2 \times 8 = 200\pi$, so $8r^2 = 200$ and $r^2 = 25$. Therefore $r = 5 \text{ cm}$.

E Example 5: unknown radius of a sphere

A sphere has volume $36\pi \text{ in}^3$. Then $\frac{4}{3}\pi r^3 = 36\pi$, so $r^3 = 36 \times \frac{3}{4} = 27$. Since $3^3 = 27$, the radius is 3 in .

7. Composite Solids

A **composite solid** is built by joining or removing simple solids. Split it into pieces you know, find each volume, then add (when pieces are joined) or subtract (when a piece is removed).



E**Example 6: cylinder topped by a hemisphere**

The toy silo above has a cylinder with $r = 3$ in and $h = 7$ in, topped by a hemisphere of the same radius.

Cylinder: $\pi \times 3^2 \times 7 = 63\pi$. Hemisphere: $\frac{2}{3}\pi \times 3^3 = 18\pi$. Total:
 $63\pi + 18\pi = 81\pi \text{ in}^3 \approx 254.3 \text{ in}^3$.

8. Real-World Volume Problems

Volume answers real questions: how much a tank holds, how much material is needed, how long it takes to fill a container. Always keep the units consistent, and convert only at the end when you can.

E**Example 7: a water tank**

A cylindrical tank has radius **2 ft** and height **5 ft**. Its volume is
 $\pi \times 2^2 \times 5 = 20\pi \approx 62.8 \text{ ft}^3$. Since $1 \text{ ft}^3 \approx 7.48$ gallons, the tank holds about
 $62.8 \times 7.48 \approx 470$ gallons.

When a solid is dropped into water, the water level rises by the volume of the solid. When a metal object is melted and recast, its volume stays the same. These two ideas turn many word problems into one volume equation.

Key takeaways

- Cylinder: $V = \pi r^2 h$. Cone: $V = \frac{1}{3} \pi r^2 h$. Sphere: $V = \frac{4}{3} \pi r^3$.
- A cone with the same base and height as a cylinder has exactly one third of its volume.
- Always use the radius, never the diameter, and measure height at a right angle to the base.
- To find an unknown dimension, substitute, isolate, then use a square root or a cube root.
- For composite solids, add the volumes of joined parts and subtract the volumes of removed parts.
- Volume is measured in cubic units; $1 \text{ cm}^3 = 1 \text{ mL}$ and $1 \text{ L} = 1,000 \text{ cm}^3$.



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