

Vectors, Polar Coordinates, and Complex Numbers: practice solutions, Grade 12 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Magnitude of a vector



Use $\|\langle a, b \rangle\| = \sqrt{a^2 + b^2}$.

a. $\sqrt{25 + 144} = \sqrt{169} = 13$.

b. $\sqrt{64 + 36} = \sqrt{100} = 10$.

c. $\sqrt{49 + 576} = \sqrt{625} = 25$.

2 Direction angles



a. Magnitude $\sqrt{1 + 3} = 2$; $\tan \theta = \sqrt{3}$ in quadrant I, so $\theta = 60^\circ$.

b. Magnitude $\sqrt{8} = 2\sqrt{2}$; $\tan \theta = 1$ but the vector is in quadrant III, so $\theta = 225^\circ$.

c. Magnitude $\sqrt{9 + 27} = 6$; $\tan \theta = -\sqrt{3}$ in quadrant IV, so $\theta = 300^\circ$.

3 Combining vectors



$$\mathbf{u} + \mathbf{v} = \langle 2, 5 \rangle. \mathbf{u} - \mathbf{v} = \langle 6, -7 \rangle. 3\mathbf{u} - 2\mathbf{v} = \langle 12, -3 \rangle - \langle -4, 12 \rangle = \langle 16, -15 \rangle.$$

4 Computing dot products



- a. $-6 + 20 = 14$; not perpendicular.
- b. $6 - 6 = 0$; the vectors are perpendicular.
- c. $-2 - 2 = -4$; not perpendicular (the vectors point in opposite directions).

5 Polar to rectangular



- a. $x = 8 \cos 60^\circ = 4$, $y = 8 \sin 60^\circ = 4\sqrt{3}$: $(4, 4\sqrt{3})$.
- b. $x = 5 \cos \pi = -5$, $y = 5 \sin \pi = 0$: $(-5, 0)$.
- c. $x = 6 \cos 270^\circ = 0$, $y = 6 \sin 270^\circ = -6$: $(0, -6)$.

6 Rectangular to polar



- a. On the positive y -axis: $(9, \frac{\pi}{2})$.
- b. On the negative x -axis: $(4, \pi)$.
- c. $r = \sqrt{18} = 3\sqrt{2}$, $\tan \theta = 1$ in quadrant I: $(3\sqrt{2}, \frac{\pi}{4})$.

7 Polar form of complex numbers



- a. $r = \sqrt{8} = 2\sqrt{2}$, the point is on the line $y = x$ in quadrant I: $2\sqrt{2} \text{ cis } 45^\circ$.
- b. On the negative real axis: $6 \text{ cis } 180^\circ$.
- c. On the negative imaginary axis: $3 \text{ cis } 270^\circ$.

8 True or false?



- a. False. With $\mathbf{u} = \langle 1, 0 \rangle$ and $\mathbf{v} = \langle 0, 1 \rangle$: $\|\mathbf{u} + \mathbf{v}\| = \sqrt{2}$ but $1 + 1 = 2$.
- b. False. $\langle 1, 0 \rangle \cdot \langle 0, 1 \rangle = 0$ and neither vector is zero; they are perpendicular.
- c. False. $(-3, \theta) = (3, \theta + \pi)$ is the point on the opposite side of the pole.

9 Kayak in a current



- a. Paddling $\langle 0, 5 \rangle$, current $\langle 3, 0 \rangle$, resultant $\langle 3, 5 \rangle$.
- b. Speed $= \sqrt{9 + 25} = \sqrt{34} \approx 5.83$ mph. In metric, $5.83 \times 1.609 \approx 9.38$ km/h.
- c. The angle from north satisfies $\tan \alpha = \frac{3}{5}$, so $\alpha \approx 31.0^\circ$ east of north (equivalently, a direction angle of 59.0° from the east axis).

The kayaker travels at about 5.83 mph, 31° east of north.

10 Pulling a sled



Force: $\mathbf{F} = \langle 50 \cos 30^\circ, 50 \sin 30^\circ \rangle = \langle 25\sqrt{3}, 25 \rangle$. Displacement: $\mathbf{d} = \langle 20, 0 \rangle$.

$$W = \mathbf{F} \cdot \mathbf{d} = 25\sqrt{3} \cdot 20 = 500\sqrt{3} \approx 866.0 \text{ ft}\cdot\text{lb (about 1,174 joules).}$$

The rope does about 866 foot-pounds of work.

11 Angle between two vectors



$$\mathbf{u} \cdot \mathbf{v} = 2 + 3 = 5, \|\mathbf{u}\| = \sqrt{5}, \|\mathbf{v}\| = \sqrt{10}.$$

$$\cos \theta = \frac{5}{\sqrt{50}} = \frac{5}{5\sqrt{2}} = \frac{\sqrt{2}}{2}, \text{ so } \theta = 45^\circ.$$

12 Polar equation to rectangular



- a. Multiply by r : $r^2 = 6r \cos \theta$, so $x^2 + y^2 = 6x$. Completing the square:
 $(x - 3)^2 + y^2 = 9$, a circle with center $(3, 0)$ and radius 3.
- b. Since $y = r \sin \theta$, the equation is $y = 4$: a horizontal line.

13 Rectangular equation to polar



- a. $r^2 = 25$, so $r = 5$.
- b. $\tan \theta = \frac{y}{x} = \sqrt{3}$, so $\theta = \frac{\pi}{3}$ (a line through the pole).
- c. $r \sin \theta = -3$, so $r = -\frac{3}{\sin \theta}$.

14 Products and quotients



$$z_1 z_2 = 4 \cdot 2 \operatorname{cis}(50^\circ + 20^\circ) = 8 \operatorname{cis} 70^\circ.$$

$$\frac{z_1}{z_2} = \frac{4}{2} \operatorname{cis}(50^\circ - 20^\circ) = 2 \operatorname{cis} 30^\circ = 2 \cos 30^\circ + 2i \sin 30^\circ = \sqrt{3} + i.$$

15 Powers with De Moivre



- a. $\cos 180^\circ + i \sin 180^\circ = -1$.
- b. $2^4 \operatorname{cis} 60^\circ = 16 \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = 8 + 8\sqrt{3}i$.

16 Cardioid and roses



(a) $r = 4, 3, 2, 1, 0$. Replacing θ by $-\theta$ does not change $\cos \theta$, so the graph is symmetric about the polar axis; its farthest point is $r = 4$ at $\theta = 0$.

(b) $n = 3$ is odd, so there are 3 petals, each of length 5 (tips at $\theta = 0, 2\pi/3, 4\pi/3$). The curve passes through the pole when $\cos 3\theta = 0$, for example at $\theta = \pi/6, \pi/2, 5\pi/6$.

(c) $n = 2$ is even, so there are $2n = 4$ petals, each of length 4.

17 Where two curves meet



Set $6 \cos \theta = 3$: $\cos \theta = \frac{1}{2}$, so $\theta = \pm \frac{\pi}{3}$. Both points have $r = 3$.

Polar: $\left(3, \frac{\pi}{3}\right)$ and $\left(3, -\frac{\pi}{3}\right)$. Rectangular: $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$ and $\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$.

18 Projection of a vector



$\mathbf{u} \cdot \mathbf{v} = 24 - 6 = 18$ and $\|\mathbf{v}\| = 5$. The scalar component is $\frac{18}{5} = 3.6$.

$\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{18}{25} \langle 4, -3 \rangle = \langle 2.88, -2.16 \rangle$.

$\mathbf{u} - \text{proj} = \langle 3.12, 4.16 \rangle$, and $3.12 \cdot 4 + 4.16 \cdot (-3) = 12.48 - 12.48 = 0$, so it is perpendicular to $\mathbf{v}$.

19 A triangle from vectors



$\overrightarrow{BA} = \langle -3, -4 \rangle$ and $\overrightarrow{BC} = \langle 4, -3 \rangle$. Their dot product is $-12 + 12 = 0$, so the angle at B is a right angle.

$\|\overrightarrow{BA}\| = \sqrt{9 + 16} = 5$ and $\|\overrightarrow{BC}\| = \sqrt{16 + 9} = 5$, so $BA = BC$: the triangle is isosceles.

Area = $\frac{1}{2} \cdot 5 \cdot 5 = 12.5$ square units.

20 A tenth power



$-1 + i = \sqrt{2} \operatorname{cis} 135^\circ$. Then $(-1 + i)^{10} = (\sqrt{2})^{10} \operatorname{cis} 1350^\circ = 32 \operatorname{cis} 270^\circ = -32i$ (since $1350^\circ - 1080^\circ = 270^\circ$).

Check: $(-1 + i)^2 = 1 - 2i - 1 = -2i$, and $(-2i)^5 = -32i^5 = -32i$. Both methods agree.

21 Sixth roots of 64



$64 = 64 \operatorname{cis} 0^\circ$, so the modulus is $\sqrt[6]{64} = 2$ and the arguments are $0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ$.

$z_0 = 2, z_1 = 1 + i\sqrt{3}, z_2 = -1 + i\sqrt{3}, z_3 = -2, z_4 = -1 - i\sqrt{3}, z_5 = 1 - i\sqrt{3}$: the vertices of a regular hexagon of radius 2.

The sum is 0, because the imaginary parts cancel in pairs and the real parts add up to $2 + 1 - 1 - 2 - 1 + 1 = 0$.

22 Fourth roots



$r = \sqrt{64 + 192} = 16$ and the point is in quadrant II with reference angle 60° , so $\theta = 120^\circ$.

The roots have modulus $\sqrt[4]{16} = 2$ and arguments $\frac{120^\circ + 360^\circ k}{4} = 30^\circ + 90^\circ k$.

$z_0 = \sqrt{3} + i, z_1 = -1 + i\sqrt{3}, z_2 = -\sqrt{3} - i, z_3 = 1 - i\sqrt{3}$.

Check: $(\sqrt{3} + i)^2 = 2 + 2\sqrt{3}i$ and $(2 + 2\sqrt{3}i)^2 = 4 - 12 + 8\sqrt{3}i = -8 + 8\sqrt{3}i$.

23

Triple-angle formulas

★★★

By the binomial theorem, with $c = \cos \theta$ and $s = \sin \theta$:

$$(c + is)^3 = c^3 + 3c^2(is) + 3c(is)^2 + (is)^3 = (c^3 - 3cs^2) + i(3c^2s - s^3).$$

De Moivre gives $\cos 3\theta + i \sin 3\theta$. Matching real parts:

$$\cos 3\theta = c^3 - 3c(1 - c^2) = 4c^3 - 3c.$$

Matching imaginary parts: $\sin 3\theta = 3(1 - s^2)s - s^3 = 3 \sin \theta - 4 \sin^3 \theta$.



Test yourself: quick challenge for Grade 12

[Change level](#)

Level : **Grade 12**

6 random questions, instant feedback.



Start the challenge



Keep exploring with Zyro



Math practice

Vectors, Polar Coordinates, and Complex Numbers: math practice, Grade 12



Math quizzes

Vectors, Polar Coordinates, and Complex Numbers: math quiz, Grade 12



Math tests

Vectors, Polar Coordinates, and Complex Numbers: math test, Grade 12



Math practice

Conic Sections and Parametric Equations: math practice, Grade 12



Math practice

Matrices and Systems of Equations: math practice, Grade 12



Math quizzes

Conic Sections and Parametric Equations: math quiz, Grade 12

Zyro takes you to our partner sites

Blast off to other planets of knowledge and their apps.

Lift off