

Vectors, Polar Coordinates, and Complex Numbers: math practice, Grade 12 – download the PDF

23 problems graded by difficulty (★ easy, ★★ medium, ★★★ hard). Try first, without looking at the solutions!

1 Magnitude of a vector



Find the magnitude of each vector.

- a. $\langle 5, 12 \rangle$
- b. $\langle -8, 6 \rangle$
- c. $\langle 7, -24 \rangle$

2 Direction angles



Find the magnitude and the direction angle (from 0° to 360°) of each vector.

- a. $\langle 1, \sqrt{3} \rangle$
- b. $\langle -2, -2 \rangle$
- c. $\langle 3, -3\sqrt{3} \rangle$

3 Combining vectors



Let $\mathbf{u} = \langle 4, -1 \rangle$ and $\mathbf{v} = \langle -2, 6 \rangle$. Compute $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$, and $3\mathbf{u} - 2\mathbf{v}$.

4 Computing dot products



Compute each dot product and say whether the vectors are perpendicular.

- a. $\langle 3, 5 \rangle \cdot \langle -2, 4 \rangle$
- b. $\langle 6, -3 \rangle \cdot \langle 1, 2 \rangle$
- c. $\langle 2, 2 \rangle \cdot \langle -1, -1 \rangle$

5 Polar to rectangular



Convert each polar point to rectangular coordinates.

- a. $\left(8, \frac{\pi}{3} \right)$
- b. $(5, \pi)$
- c. $\left(6, \frac{3\pi}{2} \right)$

6 Rectangular to polar



Give polar coordinates (r, θ) with $r > 0$ and $0 \leq \theta < 2\pi$.

- a. $(0, 9)$
- b. $(-4, 0)$
- c. $(3, 3)$

7

Polar form of complex numbers

Write each complex number in polar form $r \operatorname{cis} \theta$ with $0^\circ \leq \theta < 360^\circ$.

a. $2 + 2i$

b. -6

c. $-3i$

8

True or false?

Decide whether each statement is true or false and justify your answer with a counterexample or a short reason.

a. $\|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u}\| + \|\mathbf{v}\|$ for all vectors $\mathbf{u}, \mathbf{v}$.

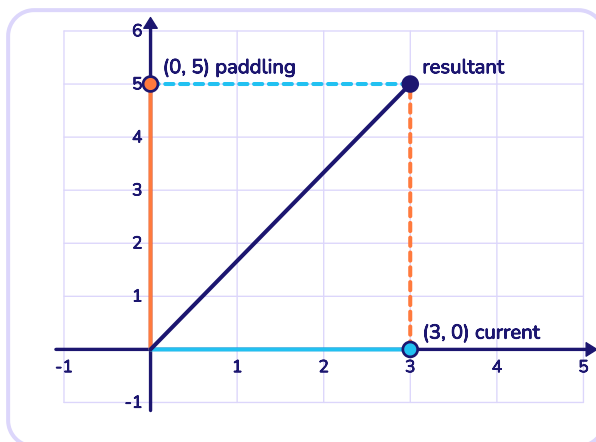
b. If $\mathbf{u} \cdot \mathbf{v} = 0$, then $\mathbf{u}$ or $\mathbf{v}$ is the zero vector.

c. The polar points $(3, \theta)$ and $(-3, \theta)$ are the same point.

9 Kayak in a current



A kayaker paddles due north at 5 mph (about 8.0 km/h) while a current flows due east at 3 mph. Take east as the positive x -direction and north as the positive y -direction.



- Write the paddling and current vectors, then the resultant velocity.
- Find the kayaker's actual speed in mph and in km/h.
- Find the direction of travel as an angle east of north.

10 Pulling a sled



A rope pulls a sled with a constant force of 50 lb directed 30° above the horizontal. The sled moves 20 ft horizontally. Find the work done, in foot-pounds.

11 Angle between two vectors



Find the angle between $\mathbf{u} = \langle 2, 1 \rangle$ and $\mathbf{v} = \langle 1, 3 \rangle$.

12 Polar equation to rectangular



Convert to rectangular form and identify the graph.

a. $r = 6 \cos \theta$

b. $r \sin \theta = 4$

13 Rectangular equation to polar



Write each equation in polar form.

a. $x^2 + y^2 = 25$

b. $y = \sqrt{3}x$

c. $y = -3$

14 Products and quotients



Let $z_1 = 4 \operatorname{cis} 50^\circ$ and $z_2 = 2 \operatorname{cis} 20^\circ$. Find $z_1 z_2$ and $\frac{z_1}{z_2}$ in polar form, then write $\frac{z_1}{z_2}$ as $a + bi$.

15 Powers with De Moivre



Use De Moivre's Theorem to evaluate.

a. $(\cos 20^\circ + i \sin 20^\circ)^9$

b. $(2 \operatorname{cis} 15^\circ)^4$, in the form $a + bi$

16 Cardioid and roses



(a) Complete a table of values for $r = 2 + 2 \cos \theta$ at $\theta = 0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi$, and state the symmetry of the graph.

(b) How many petals does $r = 5 \cos 3\theta$ have, and how long is each? (c) How many petals does $r = 4 \sin 2\theta$ have?

17 Where two curves meet



Find the points where the circles $r = 3$ and $r = 6 \cos \theta$ intersect. Give the answers in polar and rectangular coordinates.

18 Projection of a vector

★★★

Let $\mathbf{u} = \langle 6, 2 \rangle$ and $\mathbf{v} = \langle 4, -3 \rangle$. Find the scalar component of $\mathbf{u}$ along $\mathbf{v}$, the projection $\text{proj}_{\mathbf{v}} \mathbf{u}$, and verify that $\mathbf{u} - \text{proj}_{\mathbf{v}} \mathbf{u}$ is perpendicular to $\mathbf{v}$.

19 A triangle from vectors

★★★

Triangle ABC has vertices $A(1, 1)$, $B(4, 5)$, and $C(8, 2)$. Use vectors to show that the triangle has a right angle at B and is isosceles, then find its area.

20 A tenth power

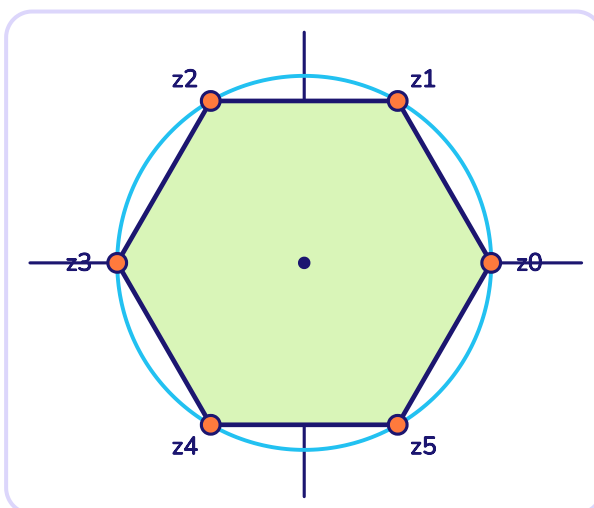
★★★

Compute $(-1 + i)^{10}$ with De Moivre's Theorem, then check your result by squaring first.

21 Sixth roots of 64

★★★

Find all complex solutions of $z^6 = 64$. Write them in the form $a + bi$, and identify the six points $z_0, \dots, z_5$ of the figure (labeled counterclockwise from the positive real axis) with these values.



What is the sum of the six roots?

22 Fourth roots



Find the four fourth roots of $-8 + 8\sqrt{3}i$ in the form $a + bi$, and check one of them by raising it to the fourth power.

23 Triple-angle formulas



Expand $(\cos \theta + i \sin \theta)^3$ with the binomial theorem and compare with De Moivre's Theorem to prove $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$. What formula for $\sin 3\theta$ do you get?



Test yourself: quick challenge for Grade 12

[Change level](#)

Level : **Grade 12**

6 random questions, instant feedback.

[Start the challenge](#)

Speed drill for Grade 12: how many in 60 seconds?

 Keep exploring with Zyro

 Practice solutions

Vectors, Polar Coordinates, and Complex Numbers: practice solutions, Grade 12

 Math lessons

Vectors, Polar Coordinates, and Complex Numbers: math lesson, Grade 12

 Math quizzes

Vectors, Polar Coordinates, and Complex Numbers: math quiz, Grade 12

 Practice solutions

Conic Sections and Parametric Equations: practice solutions, Grade 12

 Practice solutions

Matrices and Systems of Equations:

 Math lessons

Conic Sections and Parametric

practice solutions, Grade 12

Equations: math lesson, Grade 12

Zyro takes you to our partner sites

Blast off to other planets of knowledge and their apps.

Lift off