

Vector Spaces and Linear Transformations: math test solutions, College – download the PDF

Test solutions with the detailed point scale. Add up your points and spot what to review.

Suggested time: 45 minutes. Out of 20 points. Calculator allowed only when the problem says so.

1 A subspace of $\mathbb{R}^3$

/ 4 pts

- $\mathbf{0} \in W$ (0.5 pt). If $u, v \in W$, adding the equations shows $u + v \in W$ (1 pt); multiplying by c shows $cu \in W$ (0.5 pt).
- $y = 2x + z$, so $(x, y, z) = x(1, 2, 0) + z(0, 1, 1)$ (1 pt). The two vectors are independent and span W ; the dimension is 2 (1 pt).

2 Dependence

/ 3 pts

Form the matrix with these vectors as rows or columns (1 pt). Its determinant is $1(3 + 1) - 0 + 2(-2 - 0) = 0$ (1 pt). They are dependent, and $(0, -1, 3) = 2(1, 0, 2) - (2, 1, 1)$ (1 pt).

3 Null space and column space

/ 4 pts

Reduced form $\begin{pmatrix} 1 & -1 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ (1 pt). With $x_2 = s$, $x_4 = t$: $x_1 = s + 2t$, $x_3 = -t$, so

$\text{Nul}(M)$ has basis $(1, 1, 0, 0)$, $(2, 0, -1, 1)$ (1 pt). Pivots in columns 1 and 3, so $\text{Col}(M)$ has basis $(1, 2, 1)$, $(2, 5, 3)$ (1 pt). Rank 2 + nullity 2 = 4 columns (1 pt).

4 A linear map and its kernel

/ 3 pts

$A = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 1 \end{pmatrix}$ (1 pt). $2y + z = 0$ gives $z = -2y$, and $x = z - y = -3y$; a basis of $\ker T$ is $(-3, 1, -2)$ (1 pt). Rank = $3 - 1 = 2$, so T is onto (1 pt).

5 Change of basis

/ 4 pts

a. $P = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$, $\det P = 2$, $P^{-1} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix}$ (1 pt).

b. $[v]_B = \frac{1}{2}(7 - 5, -7 + 15) = (1, 4)$ (1 pt).

c. $A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$, $AP = \begin{pmatrix} 6 & 2 \\ 1 & 1 \end{pmatrix}$ (1 pt), and $[T]_B = P^{-1}AP = \begin{pmatrix} \frac{5}{2} & \frac{1}{2} \\ -\frac{3}{2} & \frac{1}{2} \end{pmatrix}$ (1 pt).

6 True or false?

/ 2 pts

(a) False (1 pt): the x -axis and y -axis are subspaces, but $(1, 0) + (0, 1) = (1, 1)$ lies in neither. (b) True (1 pt): $\dim \mathbb{R}^3 = 3$, and a family of more than 3 vectors is dependent.



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