

# Vector Spaces and Linear Transformations: math test, College – download the PDF

Suggested time: 45 minutes. Out of 20 points. Calculator allowed only when the problem says so.

## 1 A subspace of $\mathbb{R}^3$

/ 4 pts

Let  $W = \{(x, y, z) \in \mathbb{R}^3 : 2x - y + z = 0\}$ .

- Show that  $W$  is a subspace of  $\mathbb{R}^3$ .
- Find a basis of  $W$  and its dimension.

## 2 Dependence

/ 3 pts

Are  $(1, 0, 2)$ ,  $(2, 1, 1)$ ,  $(0, -1, 3)$  linearly independent? If not, give a dependence relation.

## 3 Null space and column space

/ 4 pts

Let  $M = \begin{pmatrix} 1 & -1 & 2 & 0 \\ 2 & -2 & 5 & 1 \\ 1 & -1 & 3 & 1 \end{pmatrix}$ . Find the reduced echelon form, a basis of  $\text{Nul}(M)$ , a basis of  $\text{Col}(M)$ , and check rank-nullity.

#### 4 A linear map and its kernel

/ 3 pts

Let  $T(x, y, z) = (x + y - z, 2y + z)$ . Give the matrix of  $T$ , a basis of  $\ker T$ , and decide whether  $T$  is onto  $\mathbb{R}^2$ .

#### 5 Change of basis

/ 4 pts

Let  $B = \{(3, 1), (1, 1)\}$  and  $v = (7, 5)$ .

- Write  $P$  and  $P^{-1}$ .
- Find  $[v]_B$ .
- Let  $T(x, y) = (2x, y)$ . Find  $[T]_B$ .

#### 6 True or false?

/ 2 pts

True or false? Justify. (a) The union of two subspaces of  $\mathbb{R}^2$  is always a subspace. (b) Any four vectors in  $\mathbb{R}^3$  are linearly dependent.



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