

# Vector Spaces and Linear Transformations: math practice, College – download the PDF

21 problems graded by difficulty (★ easy, ★★ medium, ★★★ hard). Try first, without looking at the solutions!

## 1 A line through the origin

★ ★ ★

Let  $W = \{(x, y) \in \mathbb{R}^2 : y = 3x\}$ . Is  $W$  a subspace of  $\mathbb{R}^2$ ? Justify.

## 2 A half-plane

★ ★ ★

Let  $H = \{(x, y) \in \mathbb{R}^2 : x \geq 0\}$ . Show that  $H$  is not a subspace.

## 3 Two pairs of vectors

★ ★ ★

Decide whether each pair is linearly independent: (a)  $(1, 2)$  and  $(2, 4)$ ; (b)  $(1, 2)$  and  $(3, 1)$ .

## 4 Counting dimensions

★ ★ ★

Give the dimension of each space: (a)  $\mathbb{R}^4$ ; (b)  $P_2$ , the polynomials of degree at most 2; (c)  $M_{2 \times 3}$ ; (d) the diagonal  $3 \times 3$  matrices.

### 5 Is it linear?



(a) Show that  $T(x, y) = (2x + y, x - y)$  is linear and find its matrix. Compute  $T(1, 2)$ . (b) Show that  $S(x, y) = (x + 1, y)$  is not linear.

### 6 Rank-nullity quick check



A  $4 \times 6$  matrix has rank 3. Find the dimension of its null space and say in which  $\mathbb{R}^k$  its column space lives.

### 7 Coordinates in a basis



Let  $B = \{(1, 0), (1, 1)\}$ . Find the coordinates of  $v = (4, 3)$  in  $B$ .

### 8 Basis of a solution set



Let  $W = \{(x, y, z) : x + y + z = 0 \text{ and } x - z = 0\}$ . Find a basis of  $W$  and its dimension.

### 9 An affine plane



Explain why  $\{(x, y, z) : x + y + z = 1\}$  is not a subspace of  $\mathbb{R}^3$ , in two different ways.

### 10 Polynomial basis



Show that  $p_1 = 1 + x$ ,  $p_2 = x + x^2$ ,  $p_3 = 1 + x^2$  form a basis of  $P_2$ .

### 11 In the span or not?



Let  $u = (1, 1, 1)$  and  $w = (0, 1, 2)$ . Decide whether (a)  $(3, 5, 7)$  and (b)  $(3, 5, 8)$  lie in  $\text{span}\{u, w\}$ .

**12 Null and column space**

Let  $B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$ . Find a basis of  $\text{Nul}(B)$  and of  $\text{Col}(B)$ , and check rank-nullity.

**13 A rotation**

The map  $T$  rotates the plane by  $90^\circ$  counterclockwise. Find its matrix, compute  $T(3, 5)$ , then  $T(T(3, 5))$ .

**14 Kernel and range**

Let  $T(x, y, z) = (x - y, y - z)$ . Find a basis of  $\ker T$  and decide whether  $T$  is onto  $\mathbb{R}^2$ .

**15 Trace-zero matrices**

Let  $W$  be the set of  $2 \times 2$  matrices with trace 0. Show that  $W$  is a subspace of  $M_{2 \times 2}$ , find a basis, and give  $\dim W$ .

**16 The derivative as a linear map**

Let  $D : P_3 \rightarrow P_2$  be  $D(p) = p'$ . Show that  $D$  is linear, find  $\ker D$ , and use rank-nullity to show  $D$  is onto.

**17 Coordinates in a new basis**

Let  $B = \{(1, 1), (1, -1)\}$ . Write  $P$ , compute  $P^{-1}$ , and find  $[v]_B$  for  $v = (7, 3)$ .

### 18 Matrix in a basis

★★★

Let  $T(x, y) = (x + 2y, 3x + 2y)$  and  $B = \{(1, 1), (1, -1)\}$ . Find  $[T]_B$  and verify that trace and determinant agree with those of the standard matrix.

### 19 Preserving independence

★★★

Suppose  $v_1, v_2, v_3$  are linearly independent. Prove that  $v_1 + v_2, v_2 + v_3, v_1 + v_3$  are also linearly independent.

### 20 A parameter and dependence

★★★

For which value of  $a$  are  $(1, 1, 0), (1, 2, 1), (2, a, 1)$  linearly dependent? Give the dependence relation.

### 21 Recovering a map from a basis

★★★

A linear map  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  satisfies  $T(1, 1) = (3, 1)$  and  $T(1, -1) = (1, 5)$ . Find its standard matrix and compute  $T(4, -2)$ .



**Test yourself: quick challenge for College**

Change level

Level : College

6 random questions, instant feedback.



**Start the challenge**



**Speed drill for College: how many in 60 seconds?**



**Keep exploring with Zyro**

 **Practice solutions**

Vector Spaces and Linear  
Transformations: practice solutions,  
College

 **Math lessons**

Vector Spaces and Linear  
Transformations: math lesson, College

 **Math quizzes**

 **Practice solutions**

Vector Spaces and Linear  
Transformations: math quiz, College

Eigenvalues and Orthogonality: practice  
solutions, College

✓ **Practice solutions**

Logic, Proofs and Discrete Math: practice  
solutions, College

■ **Math lessons**

Eigenvalues and Orthogonality: math  
lesson, College

**Zyro takes you to our partner sites**

Blast off to other planets of knowledge and their apps.

Lift off