

Dependent and Independent Variables: math lesson, Grade 6 – download the PDF

The more you charge a phone, the higher its battery level climbs. The longer you ride a bike, the farther you travel. In each case, two quantities are linked, and a change in one leads to a change in the other. In this chapter you will learn to name these quantities, record them in tables, describe them with equations and show them on a graph.

1. Quantities that change together

A **variable** is a letter or a word that stands for a quantity that can take different values. The temperature of a room, the number of tickets you buy and the time you spend jogging are all variables. When two variables are connected by a rule, we say they have a **relationship**.

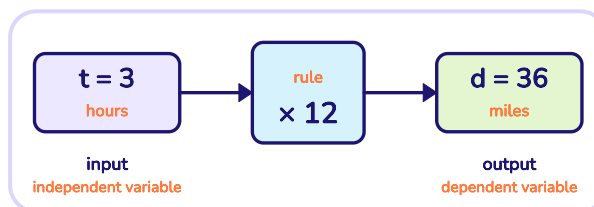
Think about a cyclist who rides at a steady pace of 12 miles every hour (about 19.3 kilometers per hour). After 1 hour she has gone 12 miles, after 2 hours 24 miles, and so on. The time and the distance change together, and the rule that links them never changes.

2. Independent and dependent variables

D Definition

The **independent variable** is the quantity you choose or that changes on its own. The **dependent variable** is the quantity that is found from it: its value depends on the independent variable.

The machine below shows the idea. You put a value of the independent variable in, the rule works on it, and a value of the dependent variable comes out.



! Zyro's tip

To decide which variable is which, finish this sentence: "The _____ depends on the _____." The quantity in the second blank is the independent variable, and the quantity in the first blank is the dependent one.

E Example 1

A car wash charges 6 dollars for each car. Let c be the number of cars washed and e the money earned, in dollars. The money earned depends on the number of cars, so c is independent and e is dependent. The relationship is $e = 6c$. For 9 cars, $e = 6 \times 9 = 54$, so the car wash earns 54 dollars.

3. Tables of values

A **table of values** lists several values of the independent variable in one row (or column) and the matching values of the dependent variable next to them. Here is the cyclist's table.

Time t (hours)	0	1	2	3	4
Distance d (miles)	0	12	24	36	48

Look at the bottom row: each number is 12 more than the one before it. Every extra hour adds the same 12 miles. Spotting this steady change is the first step toward finding the rule.

4. Equations for relationships

An **equation** in two variables is a short way to write the rule. For the cyclist, the distance is the time multiplied by 12:

$$d = 12t$$

Now you can find any distance without a table. After $t = 5$ hours, $d = 12 \times 5 = 60$ miles. You can also work backward: if she has gone 84 miles, then $12t = 84$, so $t = 84 \div 12 = 7$ hours.

5. Patterns and rules

To find the rule from a table, compare each output with its input. Ask two questions: "Was something *added*?" and "Was something *multiplied*?"

M**Method: find the rule from a table**

1. Look at how the dependent values change when the independent value goes up by 1.
2. If the dependent value goes up by the same amount each time, test an added number ($y = x + a$) or a multiplication ($y = kx$).
3. Write your guess as an equation.
4. Check the equation with **every** pair in the table, not just the first one.

E**Example 2**

Find the rule for this table.

x	1	2	3	4
y	5	6	7	8

The outputs go up by 1 each time, and $5 = 1 + 4$, $6 = 2 + 4$, $7 = 3 + 4$, $8 = 4 + 4$.

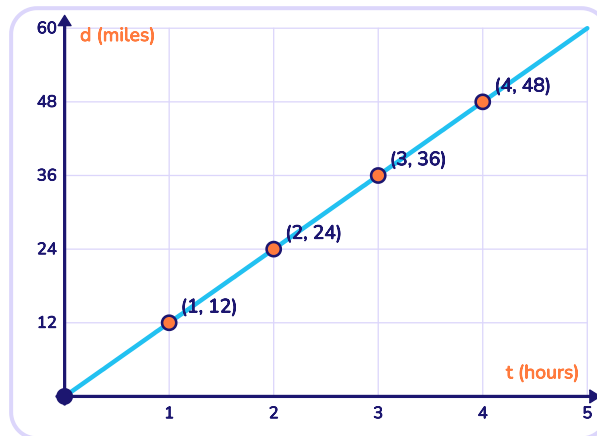
The rule is $y = x + 4$.

**Watch out**

Do not mix up adding and multiplying. The table $1, 2, 3, 4 \rightarrow 4, 8, 12, 16$ follows $y = 4x$, not $y = x + 3$. The guess $x + 3$ works for $x = 1$ only. Always test every pair.

6. Graphing relationships

Each column of a table gives an **ordered pair** (x, y) : the independent value comes first and the dependent value second. On the coordinate plane, the independent variable goes on the horizontal axis and the dependent variable on the vertical axis. For the cyclist, the pairs are $(0, 0)$, $(1, 12)$, $(2, 24)$, $(3, 36)$ and $(4, 48)$.



The points fall on a straight line that starts at the origin. A graph lets you see the whole relationship at a glance: the higher the line climbs, the faster the dependent variable grows.

7. Constant rate of change

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Definition

The **rate of change** tells how much the dependent variable changes when the independent variable increases by 1. It is **constant** when this change is always the same:

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

E**Example 3**

A pool is filled by a hose. After 2 minutes it holds 30 gallons, after 4 minutes 60 gallons and after 6 minutes 90 gallons. Between 2 and 4 minutes,

$\frac{60 - 30}{4 - 2} = \frac{30}{2} = 15$. Between 4 and 6 minutes, $\frac{90 - 60}{6 - 4} = 15$ again. The rate is a constant 15 gallons per minute (about 57 liters per minute), so $g = 15m$. After 10 minutes the pool holds $15 \times 10 = 150$ gallons.

In an equation such as $y = 12x$, the constant rate of change is the number that multiplies x . In $y = x + 4$ the rate is 1, because y goes up by 1 when x does, but the line starts at 4 instead of 0.

8. Connecting ratios to graphs

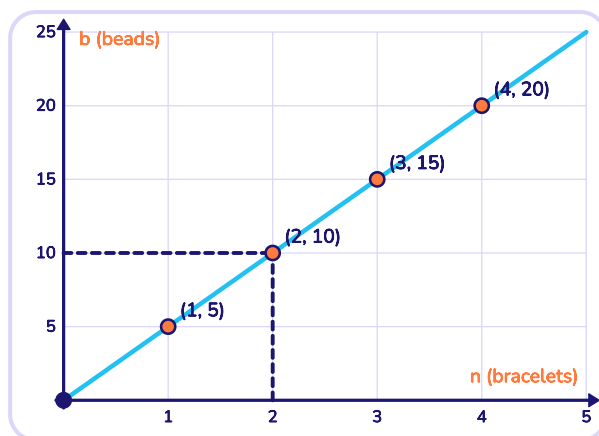
A **ratio table** is a table where every pair has the same ratio. If you graph its pairs, they always lie on a straight line through the origin $(0, 0)$, because zero of one quantity goes with zero of the other.

E**Example 4**

A jewelry maker uses 5 beads for every bracelet. The ratio of beads to bracelets is $5 : 1$.

Bracelets n	1	2	3	4
Beads b	5	10	15	20

Every pair has the ratio $5 : 1$, for example $10 : 2 = 5 : 1$. The rule is $b = 5n$ and the unit rate, 5 beads per bracelet, is the constant rate of change. In the graph, the dashed lines show the pair $(2, 10)$.



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Connection

If two quantities are in a constant ratio, their graph is a straight line through the origin, and the unit rate is the number you multiply by: $y = kx$.

Key takeaways

- The **independent variable** is the input; the **dependent variable** is the output that depends on it.
- A **table of values** pairs each input with its output, and each pair is an ordered pair (x, y) .
- An **equation** such as $y = 5x$ or $y = x + 4$ writes the rule. Test it on every pair.
- On a graph, the independent variable is on the horizontal axis and the dependent variable is on the vertical axis.
- A **constant rate of change** means the dependent variable changes by the same amount for each step of 1 in the independent variable.
- Equal ratios give points on a line through the origin, and the unit rate is the number in $y = kx$.



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