

Trigonometric Functions and the Unit Circle: practice solutions, Grade 11 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Degrees to radians



Multiply each measure by $\frac{\pi}{180}$.

a. $45 \times \frac{\pi}{180} = \frac{\pi}{4}$.

b. $120 \times \frac{\pi}{180} = \frac{2\pi}{3}$.

c. $270 \times \frac{\pi}{180} = \frac{3\pi}{2}$.

d. $150 \times \frac{\pi}{180} = \frac{5\pi}{6}$.

2 Radians to degrees



Multiply each measure by $\frac{180}{\pi}$.

a. $\frac{5\pi}{6} \times \frac{180}{\pi} = 150^\circ$.

b. $\frac{7\pi}{4} \times \frac{180}{\pi} = 315^\circ$.

c. $\frac{\pi}{9} \times \frac{180}{\pi} = 20^\circ$.

d. $\frac{11\pi}{6} \times \frac{180}{\pi} = 330^\circ$.

3 Length of a sprinkler arc



Use $s = r\theta$ with θ in radians.

$$s = 8 \times \frac{3\pi}{4} = 6\pi \approx 18.85 \text{ inches.}$$

In centimeters: $18.85 \times 2.54 \approx 47.88$ cm.

Answer: the arc is about 18.85 inches long, or about 47.9 centimeters.

4 Special values on the unit circle



These come from the first-quadrant row of the exact values table.

a. $\sin \frac{\pi}{6} = \frac{1}{2}$.

b. $\cos \frac{\pi}{3} = \frac{1}{2}$.

c. $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

d. $\cos \frac{\pi}{2} = 0$.

e. $\tan \frac{\pi}{4} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$.

5 Quadrants and signs



a. 200° is between 180° and 270° : quadrant III, sine negative, cosine negative.

b. $\frac{5\pi}{6}$ is between $\frac{\pi}{2}$ and π : quadrant II, sine positive, cosine negative.

c. 300° is between 270° and 360° : quadrant IV, sine negative, cosine positive.

d. $-\frac{\pi}{4}$ is a quarter of π clockwise, so it ends in quadrant IV: sine negative, cosine positive.

6 Amplitude and period



- a. Amplitude $|4| = 4$; period 2π because $B = 1$.
- b. Amplitude 1; period $\frac{2\pi}{3}$ because $B = 3$.
- c. Amplitude $|-2| = 2$; $B = \frac{1}{2}$, so the period is $\frac{2\pi}{1/2} = 4\pi$.

7 True or false?



- a. **True.** The point at angle π on the unit circle is $(-1, 0)$, whose x-coordinate is -1 .
- b. **False.** At $\frac{\pi}{2}$ the point is $(0, 1)$, and $\tan \frac{\pi}{2} = \frac{1}{0}$ is undefined. (The tangent equals 0 at 0 and π .)
- c. **False.** A straight angle is $180^\circ = \pi$ radians; 2π is a full turn.
- d. **True.** This is the Pythagorean identity, valid for every angle, in degrees or radians.

8 Exact values with reference angles



- a. Quadrant II, reference angle $180^\circ - 150^\circ = 30^\circ$, sine positive: $\sin 150^\circ = \frac{1}{2}$.
- b. Quadrant III, reference angle 45° , cosine negative: $\cos 225^\circ = -\frac{\sqrt{2}}{2}$.
- c. Quadrant IV, reference angle 60° , tangent negative: $\tan 300^\circ = -\sqrt{3}$.
- d. $\frac{4\pi}{3}$ is in quadrant III with reference angle $\frac{\pi}{3}$, sine negative: $\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$.

9 Cosine known, quadrant IV



By the Pythagorean identity, $\sin^2 \theta = 1 - \frac{9}{25} = \frac{16}{25}$, so $\sin \theta = \pm \frac{4}{5}$.

Sine is negative in quadrant IV, so $\sin \theta = -\frac{4}{5}$.

Then $\tan \theta = \frac{-4/5}{3/5} = -\frac{4}{3}$.

10 Sine known, quadrant II



$\cos^2 \theta = 1 - \frac{25}{169} = \frac{144}{169}$, so $\cos \theta = \pm \frac{12}{13}$.

Cosine is negative in quadrant II, so $\cos \theta = -\frac{12}{13}$.

Then $\tan \theta = \frac{5/13}{-12/13} = -\frac{5}{12}$.

11 Key points of a reflected cosine



a. Amplitude $|-3| = 3$; period $\frac{2\pi}{2} = \pi$; midline $y = 1$.

b. Maximum $1 + 3 = 4$; minimum $1 - 3 = -2$.

c. At $x = 0$: $-3(1) + 1 = -2$. At $\frac{\pi}{4}$: $-3 \cos \frac{\pi}{2} + 1 = 1$. At $\frac{\pi}{2}$: $-3 \cos \pi + 1 = 4$. At $\frac{3\pi}{4}$: $-3 \cos \frac{3\pi}{2} + 1 = 1$. At π : $-3 \cos 2\pi + 1 = -2$.

The graph starts at its minimum because of the reflection (the negative sign).

12 A sine equation



Isolate: $\sin x = \frac{1}{2}$. The reference angle is $\frac{\pi}{6}$.

Sine is positive in quadrants I and II, so $x = \frac{\pi}{6}$ or $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

13 A cosine equation



Isolate: $\cos x = -\frac{\sqrt{2}}{2}$. The reference angle is $\frac{\pi}{4}$.

Cosine is negative in quadrants II and III, so $x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ or $x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$.

14 The Ferris wheel



a. Amplitude 20 and midline 25, so the lowest height is $25 - 20 = 5$ ft and the highest is $25 + 20 = 45$ ft. The period is $\frac{2\pi}{\pi/6} = 12$ minutes. In meters, $45 \times 0.3048 \approx 13.7$ m.

b. $h(2) = 25 - 20 \cos \frac{\pi}{3} = 25 - 20 \times \frac{1}{2} = 15$ ft.

c. Solve $25 - 20 \cos \frac{\pi t}{6} = 35$: $\cos \frac{\pi t}{6} = -\frac{1}{2}$. For $0 \leq t < 12$ the angle $\frac{\pi t}{6}$ is in $[0, 2\pi)$, so $\frac{\pi t}{6} = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$, giving $t = 4$ or $t = 8$.

Answer: the seat is at 35 feet after 4 minutes and after 8 minutes.

15 Harbor tide



a. Period $\frac{2\pi}{\pi/6} = 12$ hours; maximum $8 + 3 = 11$ ft (about 3.35 m); minimum $8 - 3 = 5$ ft.

b. $d(1) = 8 + 3 \sin \frac{\pi}{6} = 8 + 1.5 = 9.5$ ft.

c. Solve $3 \sin \frac{\pi t}{6} = 1.5$, so $\sin \frac{\pi t}{6} = \frac{1}{2}$. Then $\frac{\pi t}{6} = \frac{\pi}{6}$ or $\frac{5\pi}{6}$, giving $t = 1$ or $t = 5$.

Answer: the depth is 9.5 ft at 1 a.m. and at 5 a.m.

16 A shifted sine wave

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- a. Amplitude 2, period 2π , phase shift $\frac{\pi}{3}$ to the right, range $[-2, 2]$.
- b. $\sin\left(x - \frac{\pi}{3}\right) = 0$ when $x - \frac{\pi}{3} = 0$ or π , so $x = \frac{\pi}{3}$ or $x = \frac{4\pi}{3}$.
- c. The maximum 2 occurs when $x - \frac{\pi}{3} = \frac{\pi}{2}$, so at $x = \frac{5\pi}{6}$. The minimum -2 occurs when $x - \frac{\pi}{3} = \frac{3\pi}{2}$, so at $x = \frac{11\pi}{6}$.

17 One branch of a tangent

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- a. The period is $\frac{\pi}{2}$.
- b. $\tan(2x)$ is undefined when $2x = \pm\frac{\pi}{2}$, so the asymptotes are $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$.
- c. $\tan(2x) = 0$ at $x = 0$; $\tan(2x) = 1$ when $2x = \frac{\pi}{4}$, so $x = \frac{\pi}{8}$; $\tan(2x) = -1$ when $2x = -\frac{\pi}{4}$, so $x = -\frac{\pi}{8}$. The points are $\left(-\frac{\pi}{8}, -1\right)$, $(0, 0)$ and $\left(\frac{\pi}{8}, 1\right)$.

18 Factoring a trigonometric equation

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Let $u = \cos x$. Then $2u^2 - u - 1 = 0$, which factors as $(2u + 1)(u - 1) = 0$.

So $\cos x = -\frac{1}{2}$ or $\cos x = 1$.

$\cos x = -\frac{1}{2}$: reference angle $\frac{\pi}{3}$, quadrants II and III, so $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$.

$\cos x = 1$: $x = 0$.

Solutions: $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$.

19 Using the Pythagorean identity to solve



Replace $\sin^2 x$ by $1 - \cos^2 x$: $2 - 2\cos^2 x + 3\cos x = 3$, which gives
 $2\cos^2 x - 3\cos x + 1 = 0$.

Factor: $(2\cos x - 1)(\cos x - 1) = 0$, so $\cos x = \frac{1}{2}$ or $\cos x = 1$.

$\cos x = \frac{1}{2}$: $x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$. $\cos x = 1$: $x = 0$.

Solutions: $x = 0, \frac{\pi}{3}, \frac{5\pi}{3}$.

20 Writing an equation from features



a. Amplitude $\frac{5 - (-3)}{2} = 4$; midline $\frac{5 + (-3)}{2} = 1$.

b. $B = \frac{2\pi}{\pi} = 2$. A maximum at $x = 0$ means an unreflected cosine, so
 $y = 4\cos(2x) + 1$.

c. At $x = 0$: $4\cos 0 + 1 = 5$, the maximum. At $x = \frac{\pi}{2}$: $4\cos \pi + 1 = -3$, the minimum, half a period later.

21 Daily temperature



a. The cosine equals 1 at $t = 4$, giving the minimum $62 - 14 = 48^\circ\text{F}$ at 4 a.m.; it equals -1 at $t = 16$, giving the maximum $62 + 14 = 76^\circ\text{F}$ at 4 p.m.

b. $T(10) = 62 - 14\cos \frac{\pi}{2} = 62^\circ\text{F}$.

$T(12) = 62 - 14\cos \frac{2\pi}{3} = 62 - 14 \times \left(-\frac{1}{2}\right) = 69^\circ\text{F}$.

c. Solve $62 - 14\cos u = 69$ with $u = \frac{\pi(t-4)}{12}$: $\cos u = -\frac{1}{2}$. For $0 \leq t < 24$, u runs from $-\frac{\pi}{3}$ up to just below $\frac{5\pi}{3}$, so $u = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$. Then $t - 4 = 8$ or 16 , so $t = 12$ or $t = 20$. In Celsius, $\frac{5}{9}(69 - 32) \approx 20.6^\circ\text{C}$.

Answer: 69°F (about 20.6°C) occurs at noon and at 8 p.m.

22 Proving and checking an identity



(a) By the Pythagorean identity, $\sin^2 x = 1 - \cos^2 x = (1 - \cos x)(1 + \cos x)$. Dividing by $1 - \cos x$ (not zero) gives $\frac{\sin^2 x}{1 - \cos x} = 1 + \cos x$.

(b) At $x = \frac{\pi}{3}$: $\sin^2 x = \frac{3}{4}$ and $1 - \cos x = \frac{1}{2}$, so the left side is $\frac{3/4}{1/2} = \frac{3}{2}$. The right side is $1 + \frac{1}{2} = \frac{3}{2}$. Both sides agree.



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