

Trigonometric Functions and the Unit Circle: practice solutions, Grade 12 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Degrees to radians



Multiply by $\frac{\pi}{180}$.

a. $45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$

b. $120 \cdot \frac{\pi}{180} = \frac{2\pi}{3}$

c. $270 \cdot \frac{\pi}{180} = \frac{3\pi}{2}$

d. $15 \cdot \frac{\pi}{180} = \frac{\pi}{12}$

2 Radians to degrees



Multiply by $\frac{180}{\pi}$.

a. $\frac{5 \cdot 180}{6} = 150^\circ$

b. $\frac{7 \cdot 180}{4} = 315^\circ$

c. $\frac{180}{10} = 18^\circ$

d. $\frac{3 \cdot 180}{5} = 108^\circ$

3 Length of an arc



Use $s = r\theta$ with θ in radians.

$$s = 8 \cdot \frac{\pi}{4} = 2\pi \approx 6.28 \text{ m.}$$

Answer: the path is $2\pi \approx 6.28 \text{ m}$ long (about **20.6 ft**).

4 Reading the unit circle



Use the unit circle points $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $(0, 1)$ and $(-1, 0)$.

a. $\frac{1}{2}$

b. $\frac{1}{2}$

c. $\tan \frac{\pi}{4} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

d. 0

e. 0

5 Quadrants and signs



Compare each angle with $\frac{\pi}{2}$, π , $\frac{3\pi}{2}$, 2π .

a. $\frac{\pi}{2} < \frac{5\pi}{6} < \pi$: quadrant II, **sin** > 0 , **cos** < 0 .

b. $\pi < \frac{4\pi}{3} < \frac{3\pi}{2}$: quadrant III, **sin** < 0 , **cos** < 0 .

c. $\frac{3\pi}{2} < \frac{11\pi}{6} < 2\pi$: quadrant IV, **sin** < 0 , **cos** > 0 .

d. $\frac{7\pi}{12} = 105^\circ$: quadrant II, **sin** > 0 , **cos** < 0 .

6 Right triangle ratios



Pythagorean theorem: $h^2 = 5^2 + 12^2 = 169$, so $h = 13$ in.

$$\sin \theta = \frac{5}{13}, \cos \theta = \frac{12}{13}, \tan \theta = \frac{5}{12}.$$

7 Coterminal angles



Add or subtract multiples of 2π .

a. $\frac{13\pi}{6} - 2\pi = \frac{\pi}{6}$

b. $-\frac{\pi}{3} + 2\pi = \frac{5\pi}{3}$

c. $\frac{9\pi}{4} - 2\pi = \frac{\pi}{4}$

d. $-\frac{7\pi}{4} + 2\pi = \frac{\pi}{4}$

8 Reference angles and values



a. Quadrant II, reference $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$; sine is positive: $\frac{1}{2}$.

b. Quadrant III, reference $\frac{\pi}{3}$; cosine is negative: $-\frac{1}{2}$.

c. Quadrant IV, reference $\frac{\pi}{4}$; tangent is negative: -1 .

d. Quadrant IV, reference $\frac{\pi}{6}$; cosine is positive: $\frac{\sqrt{3}}{2}$.

9 Reciprocal functions



a. $\sec \frac{\pi}{3} = \frac{1}{1/2} = 2$

b. $\sin \frac{5\pi}{6} = \frac{1}{2}$, so $\csc = 2$

c. $\cot \frac{3\pi}{4} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$

d. $\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$, so $\sec = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

10 All six from one value



$\cos^2 \theta = 1 - \frac{9}{25} = \frac{16}{25}$. In quadrant II cosine is negative, so $\cos \theta = -\frac{4}{5}$.

$\tan \theta = \frac{3/5}{-4/5} = -\frac{3}{4}$, $\csc \theta = \frac{5}{3}$, $\sec \theta = -\frac{5}{4}$, $\cot \theta = -\frac{4}{3}$.

11 A leaning ladder



The ladder is the hypotenuse.

Height: $20 \sin 70^\circ \approx 20(0.9397) \approx 18.79$ ft.

Distance: $20 \cos 70^\circ \approx 20(0.3420) \approx 6.84$ ft.

Answer: it reaches about **18.79** ft up the wall, and its foot is about **6.84** ft from the wall.

12 Sector of a pizza



Convert: $100^\circ = \frac{5\pi}{9}$.

Arc: $s = 15 \cdot \frac{5\pi}{9} = \frac{25\pi}{3} \approx 26.18$ in.

Area: $A = \frac{1}{2}(15)^2 \cdot \frac{5\pi}{9} = \frac{125\pi}{2} = 62.5\pi \approx 196.35$ in².

13 True or false?



- a. True: sine is odd.
- b. True: cosine is even.
- c. True: tangent has period π (the point at $x + \pi$ is opposite, so sine and cosine both change sign and the ratio is unchanged).
- d. False: $\sin(x + \pi) = -\sin x$. For $x = \frac{\pi}{2}$: $\sin \frac{3\pi}{2} = -1$ while $\sin \frac{\pi}{2} = 1$.

14 A bicycle wheel



$$\omega = 45 \cdot \frac{2\pi}{60} = \frac{3\pi}{2} \approx 4.71 \text{ rad/s.}$$

$$v = r\omega = 13 \cdot \frac{3\pi}{2} = 19.5\pi \approx 61.26 \text{ in/s.}$$

$$\text{In mph: } 61.26 \cdot \frac{3600}{63,360} \approx 3.48 \text{ mph.}$$

15 Solving sin equals a value



Sine is negative in quadrants III and IV. The reference angle satisfies $\sin \alpha = \frac{\sqrt{3}}{2}$, so $\alpha = \frac{\pi}{3}$.

Quadrant III: $\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$. Quadrant IV: $\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$.

Answer: $\theta = \frac{4\pi}{3}$ or $\frac{5\pi}{3}$.

16 Solving a tangent equation

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Tangent is negative in quadrants II and IV. The reference angle has $\tan \alpha = \sqrt{3}$, so $\alpha = \frac{\pi}{3}$.

Quadrant II: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$. Quadrant IV: $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$.

Answer: $\theta = \frac{2\pi}{3}$ or $\frac{5\pi}{3}$.

17 Large and negative angles

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a. $-\frac{11\pi}{6} + 2\pi = \frac{\pi}{6}$, so the value is $\frac{1}{2}$.

b. $\frac{25\pi}{4} - 6\pi = \frac{\pi}{4}$, so the value is $\frac{\sqrt{2}}{2}$.

c. The period of tangent is π : $\frac{17\pi}{3} - 5\pi = \frac{2\pi}{3}$, and $\tan \frac{2\pi}{3} = -\sqrt{3}$.

d. $-\frac{13\pi}{3} + 4\pi = -\frac{\pi}{3}$; $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$, so $\sec = 2$.

18 Finding sine and tangent from cosine

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Cosine negative and tangent positive means sine is negative too: quadrant III.

$\sin^2 \theta = 1 - \frac{25}{169} = \frac{144}{169}$, so $\sin \theta = -\frac{12}{13}$.

$\tan \theta = \frac{-12/13}{-5/13} = \frac{12}{5}$ and $\csc \theta = -\frac{13}{12}$.

19 The Ferris wheel

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$\omega = \frac{2\pi}{24} = \frac{\pi}{12} \approx 0.26 \text{ rad/min.}$

The radius is 75 ft: $v = 75 \cdot \frac{\pi}{12} = \frac{25\pi}{4} \approx 19.63 \text{ ft/min.}$

$19.63 \cdot \frac{60}{5280} \approx 0.22 \text{ mph.}$ The ride is gentle.

20 Height of a tree



The side from her eye to the horizontal at the treetop level:

$$50 \tan 36^\circ \approx 50(0.7265) \approx 36.3 \text{ ft.}$$

Add her eye height: $36.3 + 5.5 \approx 41.8 \text{ ft.}$

Answer: the tree is about **41.8** ft tall (about **12.7** m).

21 Using an identity



$\sec^2 \theta = 1 + \tan^2 \theta = 5$. Cosine (so secant) is negative in quadrant III: $\sec \theta = -\sqrt{5}$.

$$\cos \theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5} \text{ and } \sin \theta = \tan \theta \cos \theta = -\frac{2\sqrt{5}}{5}.$$

$$\text{Check: } \left(\frac{2\sqrt{5}}{5}\right)^2 = \frac{4}{5} \text{ and } \left(\frac{\sqrt{5}}{5}\right)^2 = \frac{1}{5}, \text{ and } \frac{4}{5} + \frac{1}{5} = 1.$$

22 A car tire



Speed: $60 \text{ mph} = \frac{60 \cdot 63,360}{3600} = 1056 \text{ in/s}$. Radius: **14 in**.

$$\omega = \frac{v}{r} = \frac{1056}{14} \approx 75.43 \text{ rad/s.}$$

$$\text{In rpm: } 75.43 \cdot \frac{60}{2\pi} \approx 720 \text{ rpm.}$$



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