

Trigonometric Identities and Equations: math test solutions, Grade 12 – download the PDF

Test solutions with the detailed point scale. Add up your points and spot what to review.

Suggested time: 45 minutes. Out of 20 points. Calculator allowed only when the problem says so.

1 Pythagorean identities

/ 4 pts

a. $\sec^2 \theta = 1 + \frac{64}{225} = \frac{289}{225}$; in Quadrant III cosine is negative, so $\sec \theta = -\frac{17}{15}$ and $\cos \theta = -\frac{15}{17}$. (1 pt)

b. $\sin \theta = \tan \theta \cos \theta = \frac{8}{15} \cdot \left(-\frac{15}{17}\right) = -\frac{8}{17}$. (1 pt)

c. $\sin 2\theta = 2 \left(-\frac{8}{17}\right) \left(-\frac{15}{17}\right) = \frac{240}{289}$. (1 pt)

d. $\cos 2\theta = \frac{225}{289} - \frac{64}{289} = \frac{161}{289}$. (1 pt)

2 Exact values

/ 4 pts

a. $\sin(45^\circ + 30^\circ) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$. (2 pts)

b. 22.5° is in Quadrant I, so

$$\cos 22.5^\circ = \sqrt{\frac{1 + \cos 45^\circ}{2}} = \sqrt{\frac{2 + \sqrt{2}}{4}} = \frac{\sqrt{2 + \sqrt{2}}}{2} \approx 0.924. \text{ (2 pts)}$$

3 Verifying identities

/ 4 pts

a. Multiply top and bottom of the left side by $1 + \sin x$:

$$\frac{\cos x(1 + \sin x)}{1 - \sin^2 x} = \frac{\cos x(1 + \sin x)}{\cos^2 x} = \frac{1 + \sin x}{\cos x}. \text{ (2 pts)}$$

b. $\cos 2x = \cos^2 x - \sin^2 x = (\cos x - \sin x)(\cos x + \sin x)$; dividing by $\cos x + \sin x$ leaves $\cos x - \sin x$. (2 pts)

4 A trigonometric equation

/ 4 pts

Replace $\sin^2 x$ by $1 - \cos^2 x$: $2 - 2\cos^2 x + 3\cos x = 0$, that is

$$2\cos^2 x - 3\cos x - 2 = 0. \text{ (1 pt)}$$

$$\text{Factor: } (2\cos x + 1)(\cos x - 2) = 0. \text{ (1 pt)}$$

$\cos x = 2$ is impossible because cosine never exceeds 1. (1 pt)

$$\cos x = -\frac{1}{2} \text{ gives } x = \frac{2\pi}{3} \text{ and } x = \frac{4\pi}{3}. \text{ (1 pt)}$$

5 Products and sums

/ 4 pts

a. $\sin 3x \cos x = \frac{1}{2} [\sin 4x + \sin 2x]$. (2 pts)

b. $\sin 5x - \sin x = 2 \cos 3x \sin 2x = 0$. (1 pt)

$\cos 3x = 0$ gives $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$; $\sin 2x = 0$ gives $x = 0, \frac{\pi}{2}, \pi$. Solutions:

$$0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi. \text{ (1 pt)}$$



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