

Trigonometric Identities and Equations: practice solutions, Grade 12 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Simplify with Pythagorean identities



- a. From $\sin^2 x + \cos^2 x = 1$ we get $1 - \cos^2 x = \sin^2 x$.
- b. From $1 + \tan^2 x = \sec^2 x$ we get $\sec^2 x - 1 = \tan^2 x$.
- c. From $1 + \cot^2 x = \csc^2 x$ we get $\csc^2 x - \cot^2 x = 1$.

2 Cosine given, sine and tangent wanted



$\sin^2 \theta = 1 - \frac{25}{169} = \frac{144}{169}$, so $\sin \theta = \pm \frac{12}{13}$. Sine is negative in Quadrant IV, so $\sin \theta = -\frac{12}{13}$.

Then $\tan \theta = \frac{-12/13}{5/13} = -\frac{12}{5}$.

3 Exact value of sin 15 degrees



$$\sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

.

Check: ≈ 0.259 , which is a plausible value for a small angle.

4 Cosine of a double angle



Use the form that contains only sine: $\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2 \cdot \frac{1}{9} = \frac{7}{9}$.

5 True or false: sine of a sum



False. Take $A = B = 30^\circ$. Then $\sin(A + B) = \sin 60^\circ = \frac{\sqrt{3}}{2} \approx 0.866$, but $\sin A + \sin B = \frac{1}{2} + \frac{1}{2} = 1$. One counterexample is enough to disprove it. The correct formula is $\sin A \cos B + \cos A \sin B$.

6 Exact value of $\sin 105$ degrees



$$\sin 105^\circ = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4} \approx 0.96$$

7 Tangent of a double angle



$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} = \frac{1}{3/4} = \frac{4}{3}.$$

8 A basic equation on one turn



The reference angle with sine $\frac{\sqrt{3}}{2}$ is $\frac{\pi}{3}$. Sine is positive in Quadrants I and II, so $x = \frac{\pi}{3}$ or $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

9 Cosine of 15 degrees from a difference



$$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4} = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}.$$

That is about 0.966, close to 1, as expected for a small angle.

10 Tangent of 15 degrees by half-angle



$$\tan 15^\circ = \frac{1 - \cos 30^\circ}{\sin 30^\circ} = \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3} \approx 0.268.$$

11 Half-angle with a given cosine



Since $45^\circ < \frac{\theta}{2} < 90^\circ$, both are positive.

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 + \frac{7}{25}}{2}} = \sqrt{\frac{16}{25}} = \frac{4}{5} \text{ and } \cos \frac{\theta}{2} = \sqrt{\frac{1 - \frac{7}{25}}{2}} = \sqrt{\frac{9}{25}} = \frac{3}{5}.$$

$$\text{Check: } \cos \theta = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} = \frac{9 - 16}{25} = -\frac{7}{25}.$$

12 Product-to-sum practice



a. $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$, so $2 \sin 5x \cos 3x = \sin 8x + \sin 2x$.

b. $\cos 75^\circ \cos 15^\circ = \frac{1}{2} [\cos 90^\circ + \cos 60^\circ] = \frac{1}{2} \left(0 + \frac{1}{2}\right) = \frac{1}{4}.$

13 Sum-to-product practice



a. $\sin 7x + \sin 3x = 2 \sin \frac{7x + 3x}{2} \cos \frac{7x - 3x}{2} = 2 \sin 5x \cos 2x.$

b. $\cos 75^\circ + \cos 15^\circ = 2 \cos 45^\circ \cos 30^\circ = 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2} \approx 1.225.$

14 Quadratic in sine

Factor: $(2 \sin x + 1)(\sin x - 1) = 0$, so $\sin x = -\frac{1}{2}$ or $\sin x = 1$.

$\sin x = 1$ gives $x = \frac{\pi}{2}$. $\sin x = -\frac{1}{2}$ gives angles in Quadrants III and IV: $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$.

Solutions: $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

15 Verify tan + cot

Work on the left side:

$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = \sec x \csc x$. The sides match.

16 Verify a half-angle quotient

Use $\sin 2x = 2 \sin x \cos x$ and $1 + \cos 2x = 1 + (2 \cos^2 x - 1) = 2 \cos^2 x$.

$\frac{2 \sin x \cos x}{2 \cos^2 x} = \frac{\sin x}{\cos x} = \tan x$. Verified.

17 Equation with cos 2x and cos x

Replace $\cos 2x$ by $2 \cos^2 x - 1$: $2 \cos^2 x - \cos x - 1 = 0$, so

$(2 \cos x + 1)(\cos x - 1) = 0$.

$\cos x = 1$ gives $x = 0$. $\cos x = -\frac{1}{2}$ gives $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$.

Solutions: $0, \frac{2\pi}{3}, \frac{4\pi}{3}$.

18 Do not divide by cosine



Write $2 \sin x \cos x - \sqrt{3} \cos x = 0$, then factor: $\cos x (2 \sin x - \sqrt{3}) = 0$.

$$\cos x = 0 \text{ gives } x = \frac{\pi}{2}, \frac{3\pi}{2}. \sin x = \frac{\sqrt{3}}{2} \text{ gives } x = \frac{\pi}{3}, \frac{2\pi}{3}.$$

Solutions: $\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}$. Dividing by $\cos x$ would have lost two of them.

19 Launch angle of a soccer ball



$$30 = \frac{400 \sin 2\theta}{9.8}, \text{ so } \sin 2\theta = \frac{30 \cdot 9.8}{400} = 0.735.$$

With a calculator, $2\theta \approx 47.3^\circ$ or $2\theta \approx 180^\circ - 47.3^\circ = 132.7^\circ$.

Therefore $\theta \approx 23.7^\circ$ or $\theta \approx 66.3^\circ$: a low, fast kick and a high, slow kick travel the same distance. Their angles add up to 90° .

20 Two electrical signals



Use the sum-to-product formula:

$$\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 2 \sin x \cos \frac{\pi}{3} = 2 \sin x \cdot \frac{1}{2} = \sin x.$$

Alternatively expand both with the sum formula: the terms $\pm \cos x \sin \frac{\pi}{3}$ cancel and $2 \sin x \cos \frac{\pi}{3} = \sin x$ remains.

21 Triple-angle formula



$$\cos 3x = \cos 2x \cos x - \sin 2x \sin x = (2 \cos^2 x - 1) \cos x - 2 \sin^2 x \cos x.$$

Replace $\sin^2 x$ by $1 - \cos^2 x$:

$$= 2 \cos^3 x - \cos x - 2 \cos x + 2 \cos^3 x = 4 \cos^3 x - 3 \cos x. \text{ Proved.}$$

22 Extraneous solutions



Squaring: $\sin^2 x + 2 \sin x \cos x + \cos^2 x = 1$, so $1 + \sin 2x = 1$ and $\sin 2x = 0$.

Candidates: $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$.

Check in the original equation: $x = 0$: $0 + 1 = 1$ yes. $x = \frac{\pi}{2}$: $1 + 0 = 1$ yes. $x = \pi$:

$0 - 1 = -1$ no. $x = \frac{3\pi}{2}$: $-1 + 0 = -1$ no.

Solutions: 0 and $\frac{\pi}{2}$. Squaring can introduce false solutions, so always check.



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