

Graphs of Trigonometric Functions: practice solutions, Grade 12 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Amplitude and period



In $y = A \sin(Bx)$, the amplitude is $|A| = 4$ and the period is $\frac{2\pi}{|B|} = \frac{2\pi}{3}$.

The maximum is 4 and the minimum is -4 because there is no vertical shift.

2 A reflected cosine



The amplitude is $|-2| = 2$. Here $B = \frac{1}{2}$, so the period is $\frac{2\pi}{1/2} = 4\pi$.

Because of the minus sign, the graph is reflected: at $x = 0$ we get $y = -2$, a minimum. The maximum $+2$ occurs half a period later, at $x = 2\pi$ (check: $\cos \pi = -1$, so $y = -2 \cdot (-1) = 2$).

3 Midline, maximum, minimum



The midline is $y = 5$. The amplitude is 2 , so the maximum is $5 + 2 = 7$ and the minimum is $5 - 2 = 3$.

Sine is largest at $x = \frac{\pi}{2}$ (value 7) and smallest at $x = \frac{3\pi}{2}$ (value 3).

4 A table of values



Use $\cos 0 = 1$, $\cos \frac{\pi}{3} = \frac{1}{2}$, $\cos \frac{\pi}{2} = 0$, $\cos \frac{2\pi}{3} = -\frac{1}{2}$, $\cos \pi = -1$ and multiply by 3:

x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
y	3	1.5	0	-1.5	-3

The points describe half a cycle of a cosine wave going from its maximum 3 down to its minimum -3.

5 True or false?



- True: $\cos(x + 2\pi) = \cos x$ and no smaller positive number works.
- False: tangent takes every real value, for example $\tan \frac{\pi}{4} = 1$ and $\tan \frac{\pi}{3} = \sqrt{3} \approx 1.73$.
- False: $\sec x = \frac{1}{\cos x}$ is undefined when $\cos x = 0$, for example at $x = \frac{\pi}{2}$.
- True: sine is odd, $\sin(-x) = -\sin x$, so the graph is symmetric about the origin.
- False: an amplitude is a distance and is never negative; it is $|-4| = 4$ (the minus sign only reflects the graph).

6 Asymptotes of a tangent



The period is $\frac{\pi}{|B|} = \frac{\pi}{2}$.

Asymptotes occur where $2x = \frac{\pi}{2} + k\pi$, i.e. $x = \frac{\pi}{4} + \frac{k\pi}{2}$. Taking $k = 0$ gives $\frac{\pi}{4}$; $k = -1$ gives $-\frac{\pi}{4}$; $k = 1$ gives $\frac{3\pi}{4}$ and $k = -2$ gives $-\frac{3\pi}{4}$, both outside the interval. So the asymptotes are $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$.

7 Inverse values



a. The angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with sine $\frac{1}{2}$ is $\frac{\pi}{6}$.

b. The angle in $[0, \pi]$ with cosine $-\frac{1}{2}$ is $\frac{2\pi}{3}$.

c. $\tan \frac{\pi}{4} = 1$, so $\arctan 1 = \frac{\pi}{4}$.

d. $\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ and $-\frac{\pi}{4}$ lies in the range, so the answer is $-\frac{\pi}{4}$.

8 Cosine becomes sine



Writing $x + \frac{\pi}{2} = x - \left(-\frac{\pi}{2}\right)$, the phase shift is $-\frac{\pi}{2}$: the graph of sine is slid $\frac{\pi}{2}$ units to the **left**.

The result starts at its maximum 1 when $x = 0$, so it is $y = \cos x$. For instance, at $x = \frac{\pi}{3}$:
 $\sin \frac{5\pi}{6} = \frac{1}{2} = \cos \frac{\pi}{3}$.

9 Factor before you read



Factor: $3x + \pi = 3\left(x + \frac{\pi}{3}\right)$, so $y = -4\cos\left(3\left(x + \frac{\pi}{3}\right)\right) + 2$.

Amplitude 4; period $\frac{2\pi}{3}$; phase shift $\frac{\pi}{3}$ to the **left**; midline $y = 2$; maximum $2 + 4 = 6$; minimum $2 - 4 = -2$.

At $x = 0$: $y = -4\cos \pi + 2 = -4(-1) + 2 = 6$, the maximum, which is consistent.

10 Equation of a sine wave



$A = 6, D = 2, B = \frac{2\pi}{10} = \frac{\pi}{5}$. The wave starts on the midline going up, so
 $y = 6 \sin\left(\frac{\pi x}{5}\right) + 2$.

At $x = 2.5$: $\sin \frac{\pi}{2} = 1$, so $y = 8$ (maximum). At $x = 7.5$: $\sin \frac{3\pi}{2} = -1$, so $y = -4$ (minimum).

11 A cosine from its extremes



$A = \frac{7 - (-1)}{2} = 4$ and $D = \frac{7 + (-1)}{2} = 3$. The period is $\frac{5\pi}{4} - \frac{\pi}{4} = \pi$, so $B = 2$. A maximum at $x = \frac{\pi}{4}$ means $C = \frac{\pi}{4}$.

$y = 4 \cos\left(2\left(x - \frac{\pi}{4}\right)\right) + 3$. Check: at $x = \frac{\pi}{4}$, $y = 4 + 3 = 7$; at $x = \frac{3\pi}{4}$,
 $y = 4 \cos \pi + 3 = -1$.

12 Solving trigonometric equations



a. Reference angle $\frac{\pi}{6}$, sine positive in quadrants I and II: $x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$.

b. $\cos x = -\frac{\sqrt{3}}{2}$: reference angle $\frac{\pi}{6}$, cosine negative in quadrants II and III: $x = \frac{5\pi}{6}$ or $\frac{7\pi}{6}$.

c. Reference angle $\frac{\pi}{4}$, tangent negative in quadrants II and IV: $x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$.

13 A shifted tangent



Period: $\frac{\pi}{1/2} = 2\pi$. Asymptotes: $\frac{x}{2} = \frac{\pi}{2} + k\pi$, i.e. $x = \pi + 2k\pi$; in $(-2\pi, 2\pi)$ these are $x = -\pi$ and $x = \pi$.

At $x = 0$: $y = 0 + 1 = 1$. At $x = \frac{\pi}{2}$: $y = \tan \frac{\pi}{4} + 1 = 2$.

Intercept: $\tan \frac{x}{2} = -1 \Rightarrow \frac{x}{2} = -\frac{\pi}{4} \Rightarrow x = -\frac{\pi}{2}$.

14 A stretched secant



(a) $y = \frac{2}{\cos x}$ is defined when $\cos x \neq 0$: $x \neq \frac{\pi}{2} + k\pi$. Since $|\cos x| \leq 1$, $|y| \geq 2$: the range is $(-\infty, -2] \cup [2, \infty)$.

(b) $y\left(\frac{\pi}{3}\right) = \frac{2}{1/2} = 4$ and $y\left(\frac{2\pi}{3}\right) = \frac{2}{-1/2} = -4$.

(c) At $x = 0$, $\cos x = 1$ so $y = 2$, a local minimum. At $x = \pi$, $\cos x = -1$ so $y = -2$, a local maximum.

15 Compositions with inverses



a. 0.3 lies in $[-1, 1]$, so the answer is **0.3**.

b. Right triangle with opposite 8 and hypotenuse 17: adjacent $= \sqrt{17^2 - 8^2} = 15$. So $\cos = \frac{15}{17}$.

c. Adjacent 5, hypotenuse 13: opposite $= 12$. So $\tan = \frac{12}{5}$.

d. Opposite 3, adjacent 4: hypotenuse **5**. So $\sin = \frac{3}{5}$.

16 A bicycle pedal



(a) Maximum $12 + 7 = 19$ in (about 48.3 cm); minimum $12 - 7 = 5$ in (about 12.7 cm).

(b) Period $= \frac{2\pi}{2.5\pi} = 0.8$ s. In a minute: $\frac{60}{0.8} = 75$ revolutions per minute.

(c) $h(0.1) = 12 + 7 \sin(0.25\pi) = 12 + 7 \cdot \frac{\sqrt{2}}{2} \approx 16.95$ in.

17 A sound wave



Amplitude: 0.4. Here $B = 2\pi \cdot 262$, so the period is $\frac{2\pi}{2\pi \cdot 262} = \frac{1}{262} \approx 0.003817$ s, about 3.82 ms.

In 0.05 s there are $0.05 \cdot 262 = 13.1$ cycles, so 13 full cycles.

18 Tide model



(a) Amplitude 2.5 ft; period $\frac{2\pi}{\pi/6} = 12$ h; maximum 8.5 ft; minimum 3.5 ft.

(b) $d(2) = 6 + 2.5 \cos \frac{\pi}{3} = 6 + 1.25 = 7.25$ ft.

(c) Solve $6 + 2.5 \cos \frac{\pi t}{6} \geq 7.75 \Leftrightarrow \cos \frac{\pi t}{6} \geq 0.7$. Around $t = 0$,

$\left| \frac{\pi t}{6} \right| \leq \arccos 0.7 \approx 0.7954$, so $|t| \leq \frac{6}{\pi}(0.7954) \approx 1.519$. The boat can float for about $2 \times 1.519 \approx 3.04$ hours.

19 Reading an equation from a graph

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(a) Maximum 4, minimum -2 : amplitude $\frac{4 - (-2)}{2} = 3$, midline $y = \frac{4 + (-2)}{2} = 1$. The curve repeats from $x = 0$ to $x = 8$: period 8.

(b) $B = \frac{2\pi}{8} = \frac{\pi}{4}$. The graph starts at a minimum, so use a negative cosine:
 $y = -3 \cos\left(\frac{\pi x}{4}\right) + 1$. Check: $y(0) = -2$, $y(2) = 1$, $y(4) = 4$.

(c) The rising midline crossing is at $x = 2$, so $C = 2$: $y = 3 \sin\left(\frac{\pi}{4}(x - 2)\right) + 1$. Check:
 $y(2) = 1$, $y(4) = 3 \sin \frac{\pi}{2} + 1 = 4$.

20 Inverse of the function, not the identity

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a. $\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$. The angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with that sine is $-\frac{\pi}{3}$. ($\frac{4\pi}{3}$ is outside the range.)

b. $\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$, and the angle in $[0, \pi]$ is $\frac{\pi}{4}$. ($-\frac{\pi}{4}$ is outside $[0, \pi]$.)

c. $\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$, so the answer is $\frac{5\pi}{6}$ in $[0, \pi]$.

d. $\tan \frac{3\pi}{4} = -1$, so the answer is $-\frac{\pi}{4}$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

In every case the input angle was outside the restricted range of the inverse, so the inverse returns the equivalent angle inside it.

21 An identity for inverse functions

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Let $\theta = \arcsin x$, so $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin \theta = x$. Then $\frac{\pi}{2} - \theta \in [0, \pi]$ and
 $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta = x$.

The only angle in $[0, \pi]$ whose cosine is x is $\arccos x$, so $\arccos x = \frac{\pi}{2} - \theta$, which gives
 $\arcsin x + \arccos x = \frac{\pi}{2}$.

Check: $\arcsin 0.6 \approx 0.6435$ and $\arccos 0.6 \approx 0.9273$; their sum is $1.5708 \approx \frac{\pi}{2}$.

22 A decreasing tangent



(a) Period $\frac{\pi}{\pi/4} = 4$.

(b) Asymptotes where $\frac{\pi x}{4} = \frac{\pi}{2} + k\pi \Rightarrow x = 2 + 4k$: in $(-4, 4)$ they are $x = -2$ and $x = 2$. The only intercept in $(-2, 2)$ is $x = 0$.

(c) $g(1) = -2 \tan \frac{\pi}{4} = -2$ and $g(-1) = -2 \tan \left(-\frac{\pi}{4}\right) = 2$.

(d) Tangent increases; multiplying by -2 reverses it, so g is decreasing on each branch (it falls from $+\infty$ to $-\infty$).

23 Daily temperature



(a) Maximum $73 + 11 = 84$ °F at $t = 15$ (3 PM); minimum $73 - 11 = 62$ °F at $t = 3$ (3 AM), since $\cos(-\pi) = -1$.

(b) $T(9) = 73 + 11 \cos \left(-\frac{\pi}{2}\right) = 73$ °F. $T(18) = 73 + 11 \cos \frac{\pi}{4} \approx 80.78$ °F, which is $\frac{80.78 - 32}{1.8} \approx 27.1$ °C.

(c) We need $\cos \frac{\pi(t-15)}{12} \geq \frac{7}{11} \approx 0.6364$, so

$|t - 15| \leq \frac{12}{\pi} \arccos \frac{7}{11} \approx \frac{12}{\pi} (0.8810) \approx 3.365$ h. Hence $11.63 \leq t \leq 18.37$: from about 11:38 AM to about 6:22 PM, roughly 6.7 hours.



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