

Graphs of Trigonometric Functions: math lesson, Grade 12 – download the PDF

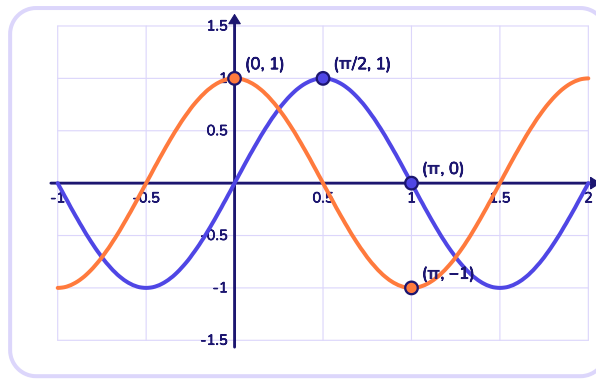
Ocean tides, the sound of a guitar string, the seat of a Ferris wheel, the length of daylight over a year: all of them repeat. In this chapter you learn to draw the graphs of the trigonometric functions, to read every feature of a wave from its equation, and to build an equation from a real situation. Angles are always measured in radians.

1. Graphs of sine and cosine

D Periodic function

A function f is **periodic** if there is a number $p > 0$ such that $f(x + p) = f(x)$ for every x in its domain. The smallest such p is the **period**.

The functions $y = \sin x$ and $y = \cos x$ are defined for every real number, take every value from -1 to 1 , and repeat every 2π . The graph of cosine is the graph of sine slid $\frac{\pi}{2}$ units to the left, because $\cos x = \sin\left(x + \frac{\pi}{2}\right)$.



P Key facts

- Domain: all real numbers. Range: $[-1, 1]$. Period: 2π .
- Sine is odd (symmetric about the origin); cosine is even (symmetric about the y -axis).
- $\sin x = 0$ at $x = k\pi$; $\cos x = 0$ at $x = \frac{\pi}{2} + k\pi$, where k is an integer.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0
$\cos x$	1	0	-1	0	1

These five points per cycle are the skeleton of every sine or cosine graph: start, quarter, half, three quarters, end.

2. Amplitude and period

Multiplying changes the wave in two different ways. For $y = A \sin(Bx)$ or $y = A \cos(Bx)$:

P**Amplitude and period**

The **amplitude** is $|A|$, half the distance between the maximum and the minimum. The **period** is $\frac{2\pi}{|B|}$. If $A < 0$, the graph is reflected over the x -axis.

E**Example 1: stretching and squeezing**

Graph one cycle of $y = 2 \sin(3x)$.

The amplitude is **2** and the period is $\frac{2\pi}{3}$. Cut the period into four equal steps of $\frac{\pi}{6}$: the key points are $(0, 0)$, $(\frac{\pi}{6}, 2)$, $(\frac{\pi}{3}, 0)$, $(\frac{\pi}{2}, -2)$ and $(\frac{2\pi}{3}, 0)$. Joining them with a smooth wave gives the graph.

M**Method: five key points**

1. Compute the period and divide it by 4 to get the horizontal step.
2. Mark the five x -values starting from the beginning of a cycle.
3. Use the pattern of the parent function (sine: 0, max, 0, min, 0; cosine: max, 0, min, 0, max), scaled by the amplitude and flipped if $A < 0$.
4. Connect with a smooth curve and repeat the cycle.

3. Phase shift and vertical shift

The general sinusoidal function is

$$y = A \sin(B(x - C)) + D.$$

P

Reading the four numbers

- $|A|$: amplitude; $\frac{2\pi}{|B|}$: period.
- C : **phase shift**, moving the graph right when $C > 0$ and left when $C < 0$.
- D : **vertical shift**; the **midline** is $y = D$, the maximum is $D + |A|$ and the minimum is $D - |A|$.

A

Factor first!

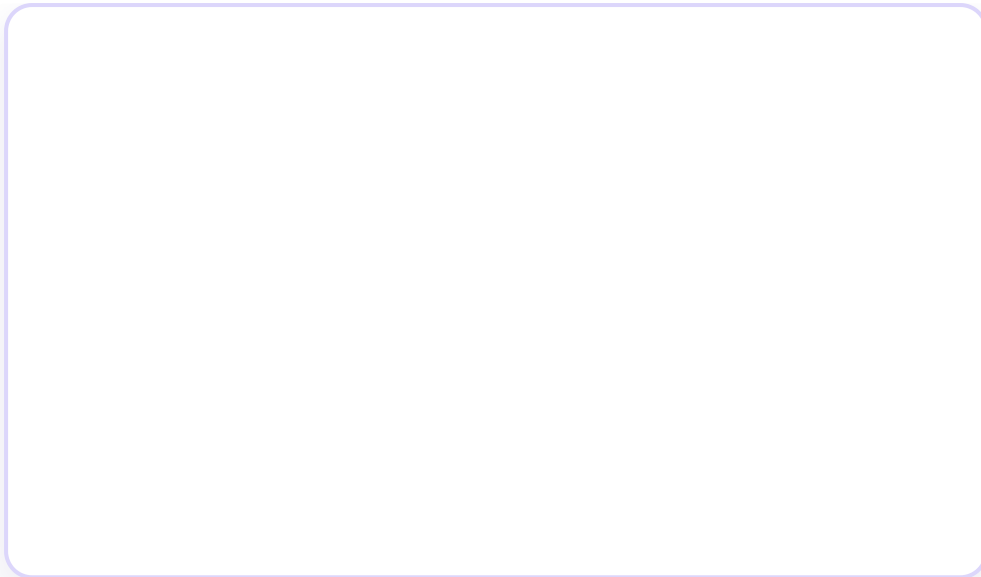
In $y = 3 \sin(2x - \frac{\pi}{2}) + 1$ the phase shift is **not** $\frac{\pi}{2}$. Factor out B :
 $2x - \frac{\pi}{2} = 2(x - \frac{\pi}{4})$, so the shift is $\frac{\pi}{4}$ to the right.

E

Example 2: all four numbers at once

Analyze $y = 3 \sin(2x - \frac{\pi}{2}) + 1$.

Rewrite it as $y = 3 \sin(2(x - \frac{\pi}{4})) + 1$. The amplitude is **3**, the period is π , the phase shift is $\frac{\pi}{4}$ to the right, and the midline is $y = 1$ with maximum **4** and minimum **-2**. The cycle starts at $x = \frac{\pi}{4}$ on the midline and the quarter-steps of $\frac{\pi}{4}$ give the points $(\frac{\pi}{4}, 1)$, $(\frac{\pi}{2}, 4)$, $(\frac{3\pi}{4}, 1)$, $(\pi, -2)$ and $(\frac{5\pi}{4}, 1)$.



Trig graphs grade 12: graph of $y = 3 \sin(2(x - \pi/4)) + 1$ with midline $y = 1$, maximum 4, minimum -2 , period π and phase shift $\pi/4$ compared with the dashed curve $y = \sin x$

! Zyro's tip

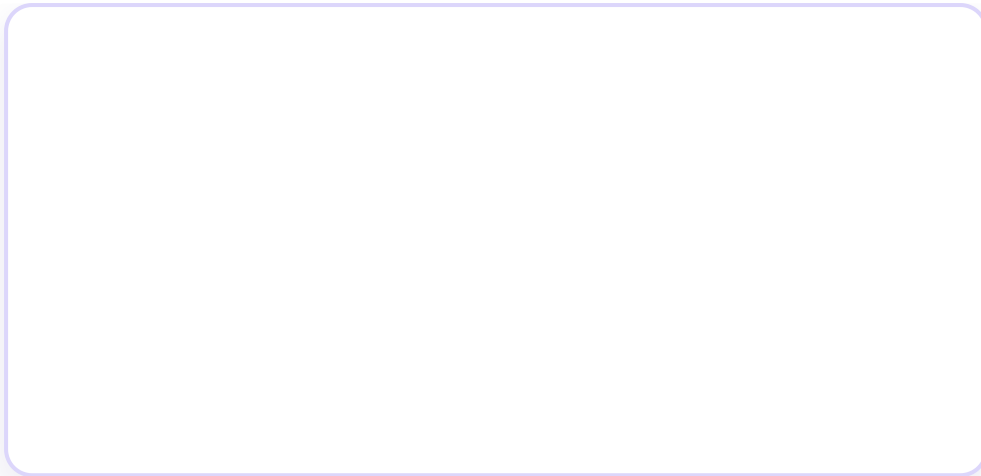
On my planet we say a wave has a “fingerprint”: midline, amplitude, period, start. Find those four things and you can draw the wave with your eyes closed!

4. Graphs of tangent and cotangent

D Tangent and cotangent

$\tan x = \frac{\sin x}{\cos x}$ and $\cot x = \frac{\cos x}{\sin x}$, defined wherever the denominator is not zero.

- $y = \tan x$: period π ; range all real numbers; vertical asymptotes at $x = \frac{\pi}{2} + k\pi$; zeros at $x = k\pi$; always increasing on each branch.
- $y = \cot x$: period π ; range all real numbers; vertical asymptotes at $x = k\pi$; zeros at $x = \frac{\pi}{2} + k\pi$; always decreasing on each branch.



Trig graphs grade 12: graphs of tangent and cotangent with period π , dashed vertical asymptotes and branches that climb from minus infinity to plus infinity for tangent and fall for cotangent

For $y = A \tan(Bx)$ the period is $\frac{\pi}{|B|}$. Tangent has no maximum, so $|A|$ is not an amplitude: it stretches the branches vertically.

E Example 3: a squeezed tangent

Describe $y = \tan(2x)$.

The period is $\frac{\pi}{2}$. The asymptotes solve $2x = \frac{\pi}{2} + k\pi$, that is $x = \frac{\pi}{4} + \frac{k\pi}{2}$; the two closest to the origin are $x = \pm \frac{\pi}{4}$. The graph crosses the x -axis at $x = 0$, and since $\tan \frac{\pi}{4} = 1$, it passes through $(\frac{\pi}{8}, 1)$ and $(-\frac{\pi}{8}, -1)$.

5. Graphs of secant and cosecant

D Reciprocal functions

$\sec x = \frac{1}{\cos x}$ and $\csc x = \frac{1}{\sin x}$. Both have period 2π .

M**Method: sketch from the parent wave**

1. Draw the cosine (or sine) curve lightly.
2. Draw a vertical asymptote at every zero of that curve.
3. Where the curve has a maximum or minimum, draw a U-shaped branch touching it, opening away from the x -axis.



Trig graphs grade 12: graphs of secant and cosecant drawn as U-shaped branches that touch the dashed cosine and sine curves at their maximum and minimum points, with asymptotes where those curves cross zero

Since $|\cos x| \leq 1$, we get $|\sec x| \geq 1$: the range of secant and cosecant is $(-\infty, -1] \cup [1, \infty)$.
Secant has asymptotes at $x = \frac{\pi}{2} + k\pi$, cosecant at $x = k\pi$.

6. Inverse trigonometric functions

Sine, cosine and tangent repeat their values, so they are not one-to-one. We restrict each to an interval where it is, and then invert it.

Function	Domain	Range (output angle)
$y = \arcsin x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \arccos x$	$[-1, 1]$	$[0, \pi]$
$y = \arctan x$	all real numbers	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Trig graphs grade 12: graphs of arcsine, arccosine and arctangent showing their restricted ranges and the horizontal asymptotes $y = \pi/2$ and $y = -\pi/2$ of arctangent



Cancellation rules

- $\sin(\arcsin x) = x$ for $-1 \leq x \leq 1$, but $\arcsin(\sin x) = x$ only when $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.
- $\arccos(\cos x) = x$ only when $0 \leq x \leq \pi$; $\arctan(\tan x) = x$ only when $-\frac{\pi}{2} < x < \frac{\pi}{2}$.



Not a reciprocal

arcsin x means “the angle whose sine is x ”. It is not $\frac{1}{\sin x}$, which is **csc** x .

E**Example 4: two inverse evaluations**

(a) $\arccos\left(\cos \frac{5\pi}{4}\right)$: $\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$, and the angle in $[0, \pi]$ with that cosine is $\frac{3\pi}{4}$. So the answer is $\frac{3\pi}{4}$, not $\frac{5\pi}{4}$.

(b) $\cos\left(\arcsin \frac{3}{5}\right)$: let $\theta = \arcsin \frac{3}{5}$. In a right triangle with opposite side 3 and hypotenuse 5, the adjacent side is 4, so $\cos \theta = \frac{4}{5}$.

7. Modeling periodic phenomena**M****Method: from data to equation**

1. Amplitude $A = \frac{\text{max} - \text{min}}{2}$ and midline $D = \frac{\text{max} + \text{min}}{2}$.
2. Find the period P and set $B = \frac{2\pi}{P}$.
3. Choose cosine if you know where a maximum or minimum occurs (use $-\cos$ if time 0 is a minimum), sine if the cycle starts on the midline going up.
4. Check the model against one known data point, then answer the question.

E

Example 5: a Ferris wheel

A wheel has diameter 80 feet (about 24.4 m), its center is 50 feet (15.2 m) above the ground, and it turns once every 6 minutes. A rider boards at the lowest point.

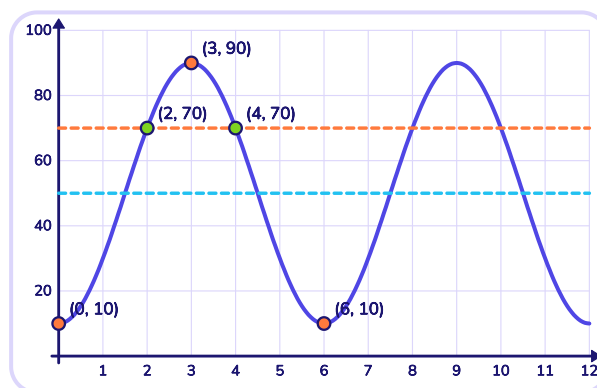
The minimum is $50 - 40 = 10$ feet, the maximum is 90 feet, so $A = 40$ and $D = 50$.

The period is 6 minutes, so $B = \frac{2\pi}{6} = \frac{\pi}{3}$. Starting at a minimum gives

$$h(t) = 50 - 40 \cos\left(\frac{\pi t}{3}\right).$$

Check: $h(0) = 10$. To find when the rider is higher than 70 feet, solve

$50 - 40 \cos \frac{\pi t}{3} \geq 70$, i.e. $\cos \frac{\pi t}{3} \leq -\frac{1}{2}$. On one turn this means $\frac{2\pi}{3} \leq \frac{\pi t}{3} \leq \frac{4\pi}{3}$, so $2 \leq t \leq 4$: two minutes per turn.



Key takeaways

- Sine and cosine have period 2π , range $[-1, 1]$; cosine is sine shifted $\frac{\pi}{2}$ to the left.
- For $y = A \sin(B(x - C)) + D$: amplitude $|A|$, period $\frac{2\pi}{|B|}$, phase shift C , midline $y = D$. Always factor B before reading C .
- Tangent and cotangent have period $\frac{\pi}{|B|}$ after the change $x \rightarrow Bx$, vertical asymptotes, and range all reals.
- Secant and cosecant are reciprocals: asymptotes at the zeros of cosine or sine, range $(-\infty, -1] \cup [1, \infty)$ before stretching.
- Inverse functions need restricted ranges: **arcsin** and **arctan** in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, **arccos** in $[0, \pi]$.
- To model a cycle: $A = \frac{\max - \min}{2}$, $D = \frac{\max + \min}{2}$, $B = \frac{2\pi}{\text{period}}$.



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