

Relationships in Triangles: practice solutions, Grade 10 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Length of a midsegment



By the Midsegment Theorem, the midsegment is half as long as the third side:

$$DE = \frac{1}{2}(34) = 17 \text{ in.}$$

Also, $\overline{DE} \parallel \overline{BC}$.

Answer: $DE = 17$ in, and $\overline{DE}$ is parallel to $\overline{BC}$.

2 Doubling a midsegment



The side parallel to a midsegment is twice as long as the midsegment: $2 \times 9.5 = 19$.

Answer: the parallel side measures **19** cm.

3 Equal distances from a perpendicular bisector



A point on the perpendicular bisector is equidistant from the endpoints, so $PX = PY$.

$$2x + 5 = 4x - 7, \text{ so } 12 = 2x \text{ and } x = 6.$$

$$PX = 2(6) + 5 = 17. \text{ Check: } PY = 4(6) - 7 = 17.$$

Answer: $x = 6$ and $PX = 17$.

4 Equal distances from an angle bisector



A point on an angle bisector is the same distance from both sides, so $4y - 3 = 2y + 9$.

$2y = 12$, so $y = 6$. The distance is $4(6) - 3 = 21$ mm. Check: $2(6) + 9 = 21$.

Answer: $y = 6$ and the distance is 21 mm.

5 Splitting a median



The centroid is $\frac{2}{3}$ of the way from the vertex: $AG = \frac{2}{3}(18) = 12$ cm.

The rest of the median is $GM = 18 - 12 = 6$ cm. Check: $AG = 2GM$.

Answer: $AG = 12$ cm and $GM = 6$ cm.

6 Which center should you use?



- a. Equal distances from three points: the **circumcenter** (perpendicular bisectors).
- b. A circle tangent to the three sides is centered at the **incenter** (angle bisectors).
- c. The balance point is the **centroid** (medians).

7 Where is the orthocenter?



- a. All angles are less than 90° : acute triangle, so the orthocenter is **inside**.
- b. There is a right angle: the orthocenter is **at the vertex of the right angle**.
- c. One angle is greater than 90° : obtuse triangle, so the orthocenter is **outside**.

8 Can these lengths form a triangle?



- a. $6 + 9 = 15 > 14$: yes.
- b. $5 + 8 = 13$, which is not greater than **13**: **no** (the triangle would be flat).
- c. $4 + 4 = 8 < 9$: no.
- d. $10 + 12 = 22 > 21$: yes.

Only the two smallest lengths need to be added, because the third inequality then holds automatically.

9 Midsegment with algebra



$$BC = 2 DE, \text{ so } 12x - 26 = 2(5x - 4) = 10x - 8.$$

$$2x = 18, \text{ so } x = 9.$$

$$DE = 5(9) - 4 = 41 \text{ and } BC = 12(9) - 26 = 82. \text{ Check: } 82 = 2 \times 41.$$

$$\text{Answer: } x = 9, DE = 41, BC = 82.$$

10 The medial triangle



- a. Each side of the small triangle is a midsegment, so it is half of the side it is parallel to:
7 cm, 9 cm and 11 cm.
- b. The perimeter of ABC is $14 + 18 + 22 = 54$ cm. The small triangle has perimeter $7 + 9 + 11 = 27$ cm, which is exactly half.

11 Three trees and a fountain



a. $20^2 + 21^2 = 400 + 441 = 841 = 29^2$, so the triangle is right (converse of the Pythagorean theorem) and **29** m is the hypotenuse.

b. The point equidistant from the three vertices is the circumcenter. In a right triangle it is the midpoint of the hypotenuse. The distance is $29 \div 2 = 14.5$ m.

Answer: place the fountain at the midpoint of the **29** m side; it is **14.5** m from each tree.

12 The largest pond



Since $9^2 + 12^2 = 81 + 144 = 225 = 15^2$, the bed is a right triangle with area $\frac{1}{2}(9)(12) = 54 \text{ ft}^2$.

The largest circle that fits is the inscribed circle, centered at the incenter. Its radius is area $\div$ semiperimeter: $\frac{9 + 12 + 15}{2} = 18$, so $r = 54 \div 18 = 3$ ft.

Answer: the pond can have a radius of **3** ft (about **0.91** m).

13 Centroid coordinates



a. $x = \frac{-3 + 9 + 3}{3} = 3$ and $y = \frac{2 + 4 + 12}{3} = 6$, so $G(3, 6)$.

b. $M = \left(\frac{9 + 3}{2}, \frac{4 + 12}{2} \right) = (6, 8)$. Going two thirds of the way from A to M :
 $\left(-3 + \frac{2}{3}(9), 2 + \frac{2}{3}(6) \right) = (3, 6)$, which is G .

14 Medians with algebra



The centroid satisfies $BG = 2GE$: $5x - 7 = 2(x + 1) = 2x + 2$, so $3x = 9$ and $x = 3$.

$BG = 5(3) - 7 = 8$ and $GE = 3 + 1 = 4$, so $BE = 8 + 4 = 12$.

Answer: $x = 3$ and $BE = 12$.

15 Range for the third side

- a. $17 - 11 < x < 17 + 11$, so $6 < x < 28$.
- b. The whole numbers allowed are $7, 8, \dots, 27$. The smallest perimeter is $11 + 17 + 7 = 35$ cm and the largest is $11 + 17 + 27 = 55$ cm.

16 Shortest and longest side

Check: $48 + 75 + 57 = 180$.

$\overline{BC}$ is opposite $\angle A = 48^\circ$, $\overline{AB}$ is opposite $\angle C = 57^\circ$ and $\overline{AC}$ is opposite $\angle B = 75^\circ$.

Since $48 < 57 < 75$: $BC < AB < AC$.

17 A midsegment on the coordinate plane

- a. $M = \left(\frac{2+10}{2}, \frac{1+3}{2} \right) = (6, 2)$ and $N = \left(\frac{2+6}{2}, \frac{1+9}{2} \right) = (4, 5)$.
- b. Slope of $\overline{MN}$: $\frac{5-2}{4-6} = -\frac{3}{2}$. Slope of $\overline{BC}$: $\frac{9-3}{6-10} = -\frac{3}{2}$. Equal slopes, so the segments are parallel.
- c. $MN = \sqrt{(4-6)^2 + (5-2)^2} = \sqrt{13}$ and
 $BC = \sqrt{(6-10)^2 + (9-3)^2} = \sqrt{52} = 2\sqrt{13}$. So $MN = \frac{1}{2} BC$.

18 Finding a circumcenter

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a. $\overline{AB}$ is horizontal with midpoint $(5, 0)$, so its perpendicular bisector is the vertical line $x = 5$.

b. The midpoint of $\overline{AC}$ is $(1, 3)$ and its slope is $\frac{6}{2} = 3$. The perpendicular bisector has slope $-\frac{1}{3}$: $y - 3 = -\frac{1}{3}(x - 1)$.

c. At $x = 5$: $y = 3 - \frac{4}{3} = \frac{5}{3}$, so $O\left(5, \frac{5}{3}\right)$.

$OA^2 = 25 + \frac{25}{9} = \frac{250}{9}$ and $OC^2 = 3^2 + \left(\frac{5}{3} - 6\right)^2 = 9 + \frac{169}{9} = \frac{250}{9}$. Hence $OA = OC$.

19 Recover a missing vertex

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Let $C = (p, q)$. The centroid averages the vertices: $\frac{2 + 9 + p}{3} = 4$ and $\frac{1 + 6 + q}{3} = 5$.

So $11 + p = 12$, $p = 1$, and $7 + q = 15$, $q = 8$.

Answer: $C(1, 8)$. Check: $\frac{2 + 9 + 1}{3} = 4$ and $\frac{1 + 6 + 8}{3} = 5$.

20 Finding an orthocenter

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The side $\overline{AB}$ lies on the x-axis, so the altitude from C is $x = 2$.

The slope of $\overline{BC}$ is $\frac{4 - 0}{2 - 6} = -1$, so the altitude from A has slope 1: $y = x$. It meets $x = 2$ at $(2, 2)$.

Check: the slope of $\overline{AC}$ is 2, so the altitude from B has slope $-\frac{1}{2}$: $y = -\frac{1}{2}(x - 6)$. At $x = 2$, $y = 2$.

Answer: the orthocenter is $H(2, 2)$, inside the triangle because ABC is acute ($AB^2 = 36 < AC^2 + BC^2 = 20 + 32$).

21 An inequality in the unknown



All sides must be positive: $2x - 3 > 0$, so $x > 1.5$.

Apply the triangle inequality to each pair:

- $5 + x > 2x - 3$ gives $x < 8$;
- $5 + (2x - 3) > x$ gives $x > -2$;
- $x + (2x - 3) > 5$ gives $3x > 8$, so $x > \frac{8}{3}$.

The most restrictive conditions are $x > \frac{8}{3}$ and $x < 8$.

Answer: $\frac{8}{3} < x < 8$.

22 Angles with algebra



The angles sum to 180° : $6x + 30 = 180$, so $x = 25$.

$\angle P = 60^\circ$, $\angle Q = 70^\circ$, $\angle R = 50^\circ$.

$\overline{QR}$ is opposite P (60°), $\overline{PR}$ is opposite Q (70°) and $\overline{PQ}$ is opposite R (50°).

Answer: $PQ < QR < PR$.

23 Three lines in one



Since $AB = AC$, point A is equidistant from B and C , so by the converse of the Perpendicular Bisector Theorem A lies on the perpendicular bisector of $\overline{BC}$.

The midpoint M of $\overline{BC}$ also lies on that perpendicular bisector. Two points determine a line, so line AM is the perpendicular bisector of $\overline{BC}$.

Therefore $\overline{AM} \perp \overline{BC}$, so $\overline{AM}$ is an altitude, and it is a median by definition. In an isosceles triangle, the median, altitude and perpendicular bisector from the apex coincide.



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