

Transformations and Congruence: math lesson, Grade 8 – download the PDF

Slide a sticker across a table, flip it over, or spin it around a pin: the sticker keeps exactly the same size and shape. In this chapter you will learn the three moves that do this, how to describe them with coordinates, and how they let you prove that two figures are congruent.

1. What is a transformation?

A **transformation** takes every point of a figure, called the **preimage**, and moves it to a new position. The new figure is the **image**. We name an image point with a prime: the image of point **A** is **A'** (read “A prime”).

D Rigid transformation

A **rigid transformation** (also called a **rigid motion** or an **isometry**) moves a figure without changing its size or its shape. The three rigid transformations are the **translation**, the **reflection**, and the **rotation**.

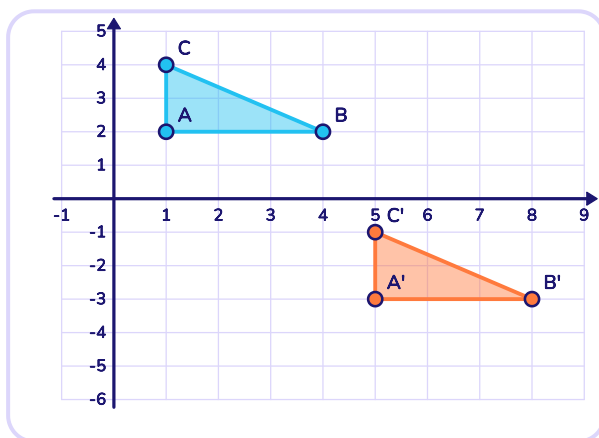
Other transformations, such as stretching a picture to make it twice as big, are *not* rigid because they change lengths.

2. Translations

D Translation

A **translation** slides every point of a figure the same distance in the same direction. It is described by a **vector** $\langle a, b \rangle$: move a units horizontally (right if positive, left if negative) and b units vertically (up if positive, down if negative).

On the coordinate plane, the point (x, y) goes to $(x + a, y + b)$.



E Example 1 : translating a triangle

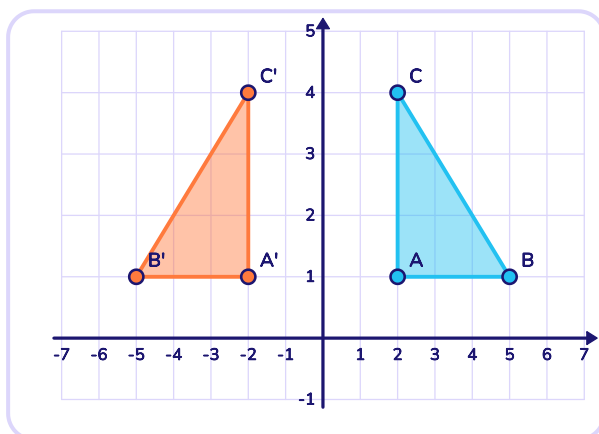
Translate $A(-2, 3)$, $B(1, 3)$, $C(-2, -1)$ by $\langle 5, -2 \rangle$.

Add 5 to every x -coordinate and subtract 2 from every y -coordinate: $A'(3, 1)$, $B'(6, 1)$, $C'(3, -3)$. Each side keeps its length: $AB = 3$ and $A'B' = 3$.

3. Reflections

D Reflection

A **reflection** across a **line of reflection** flips a figure over that line, like a mirror. Each point and its image are the same distance from the line, and the line is the perpendicular bisector of the segment joining them.



On the coordinate plane: across the **x-axis**, $(x, y) \rightarrow (x, -y)$; across the **y-axis**, $(x, y) \rightarrow (-x, y)$.

E Example 2 : reflecting a point

Reflect $P(4, -3)$ across each axis.

Across the x-axis, only the sign of **y** changes: $(4, 3)$. Across the y-axis, only the sign of **x** changes: $(-4, -3)$. In both cases the image is as far from the axis as **P** is: 3 units from the x-axis in the first case, 4 units from the y-axis in the second.



Careful with orientation

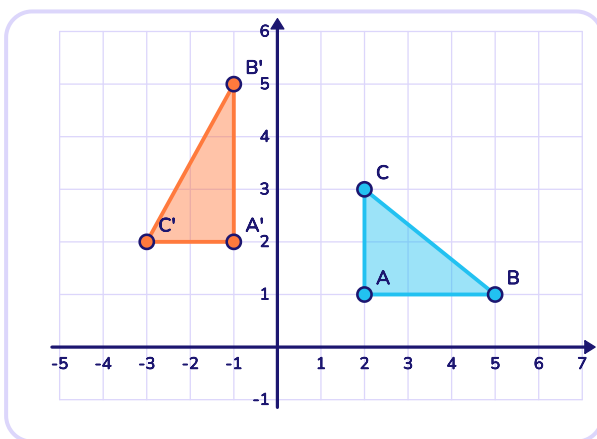
A reflection reverses the order of the vertices: if you walk $A \rightarrow B \rightarrow C$ clockwise on the original, you walk $A' \rightarrow B' \rightarrow C'$ counterclockwise on the image. The shape is a mirror image, but every length and angle is the same.

4. Rotations



Rotation

A **rotation** turns every point of a figure by the same angle around a fixed point called the **center of rotation**. We give the angle and the direction: **counterclockwise** (the direction opposite to clock hands, taken as positive) or **clockwise**. Each point stays the same distance from the center.



Rotations about the origin

90° counterclockwise: $(x, y) \rightarrow (-y, x)$. 180° : $(x, y) \rightarrow (-x, -y)$. 90° clockwise (the same as 270° counterclockwise): $(x, y) \rightarrow (y, -x)$.

E**Example 3 : rotating a point**

Rotate $M(3, -2)$ about the origin. By 90° counterclockwise: $(2, 3)$. By 180° : $(-3, 2)$. By 90° clockwise: $(-2, -3)$.

Check: M is $\sqrt{3^2 + (-2)^2} = \sqrt{13}$ units from the origin, and so is each image, as expected for a rotation about the origin.

5. Congruent figures**D****Congruence**

Two figures are **congruent**, written $\cong$, if one can be obtained from the other by a sequence of rotations, reflections, and translations. In other words, they have exactly the same size and the same shape, and only their position or orientation may differ.

When two polygons are congruent, the matching points are called **corresponding** parts. Corresponding sides have equal lengths and corresponding angles have equal measures. We write the vertices in matching order: $\triangle ABC \cong \triangle A'B'C'$ tells you that A matches A' , B matches B' , and C matches C' .

6. Properties of rigid motions

P

What a rigid motion preserves

- Lengths of segments (so perimeters are preserved too).
- Measures of angles.
- Parallel lines stay parallel, and perpendicular lines stay perpendicular.
- Areas.

What can change is the **location** of the figure and, for a reflection, its **orientation**. A line segment is always sent to a line segment of the same length, a line to a line, and a ray to a ray.

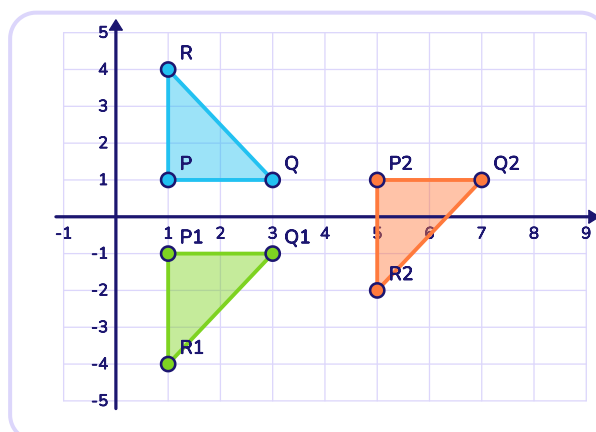
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Zyro's trick

On my home planet we test congruence with tracing paper: trace the first figure, then slide, flip, or turn the paper until it covers the second one exactly. If it fits, there is a rigid motion that does the job!

7. Sequences of transformations

You can apply one transformation after another. Because each step is rigid, the final image is still congruent to the starting figure. **The order can matter.**



E**Example 4 : two transformations in a row**

Take $A(2, 5)$. Reflect it across the y-axis, then translate by $\langle 3, 0 \rangle$:

$(2, 5) \rightarrow (-2, 5) \rightarrow (1, 5)$. Now do it in the other order: translate first,

$(2, 5) \rightarrow (5, 5)$, then reflect, $(5, 5) \rightarrow (-5, 5)$. The two results, $(1, 5)$ and $(-5, 5)$, are different.

M**Method : describe a sequence on the coordinate plane**

1. Write the rule for the first transformation and apply it to every vertex.
2. Use the new points as the starting points for the second transformation.
3. Label each stage (A , A_1 , A_2 , and so on).
4. Check one side length at the start and at the end: they must be equal.

8. Coordinates under transformations

Here is the summary of the rules you can use for any point (x, y) .

Transformation	Rule for a point (x, y)	Image of $(3, -2)$
Translation by $\langle a, b \rangle$	$(x + a, y + b)$	with $\langle 4, 1 \rangle$: $(7, -1)$
Reflection across the x-axis	$(x, -y)$	$(3, 2)$
Reflection across the y-axis	$(-x, y)$	$(-3, -2)$
Rotation 90° counterclockwise about the origin	$(-y, x)$	$(2, 3)$
Rotation 180° about the origin	$(-x, -y)$	$(-3, 2)$
Rotation 90° clockwise about the origin	$(y, -x)$	$(-2, -3)$

To show that two figures are congruent on the coordinate plane, find a sequence of these moves that sends every vertex of the first figure onto the matching vertex of the second one.

Key takeaways

- Translations, reflections, and rotations are rigid: they keep lengths, angle measures, and areas.
- Translation by $\langle a, b \rangle$: $(x, y) \rightarrow (x + a, y + b)$.
- Reflection across the x-axis: $(x, -y)$. Across the y-axis: $(-x, y)$.
- Rotation 90° counterclockwise about the origin: $(-y, x)$; 180° : $(-x, -y)$.
- Two figures are congruent if a sequence of rigid motions maps one onto the other.
- The order of the transformations in a sequence can change the final image.



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