

Rigid Motions and Transformations: math lesson, Grade 10 – download the PDF

Slide a chess piece across the board, flip a pancake, spin a Ferris wheel: in each case an object changes position, yet it keeps exactly the same size and shape. In this chapter you will describe these moves with **coordinate rules**, combine them, and use them to decide when two figures are **congruent**.

1. Transformations and rigid motions

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Transformation

A **transformation** assigns to every point of the plane a new point. The starting figure is the **preimage**; the figure you get is the **image**. We write A for a point of the preimage and A' (read “A prime”) for its image.

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Rigid motion

A **rigid motion** (also called an **isometry**) is a transformation that preserves **distance** and **angle measure**. The image has exactly the same size and shape as the preimage; only its position or its orientation may change.

Three basic rigid motions are studied here: **translations** (slides), **reflections** (flips) and **rotations** (turns). A dilation, which enlarges or shrinks a figure, is *not* a rigid motion because it changes distances.

2. Translations

D Translation

A **translation** slides every point the same distance in the same direction. It is described by a vector $\langle a, b \rangle$: move a units horizontally and b units vertically.

P Coordinate rule

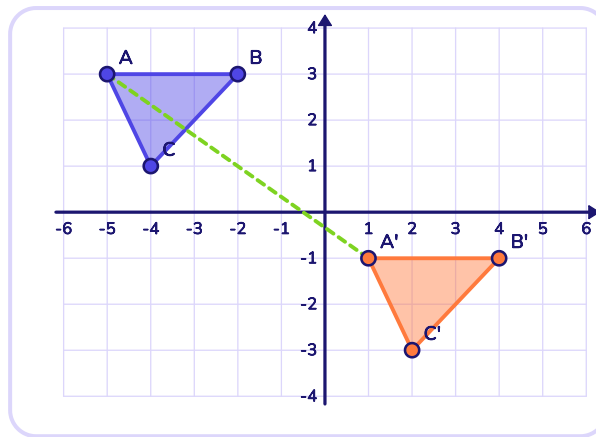
The translation by $\langle a, b \rangle$ maps (x, y) to $(x + a, y + b)$.

E Example 1 : sliding a triangle

Translate $A(-5, 3)$, $B(-2, 3)$, $C(-4, 1)$ by $\langle 6, -4 \rangle$ (6 right, 4 down).

$$A' = (-5 + 6, 3 - 4) = (1, -1), B' = (-2 + 6, 3 - 4) = (4, -1) \text{ and } C' = (-4 + 6, 1 - 4) = (2, -3).$$

Check: $AB = 3$ and $A'B' = 3$. The sliding keeps every length.



3. Reflections

D Reflection

A **reflection** in a line ℓ (the **line of reflection**) maps each point P to a point P' such that ℓ is the **perpendicular bisector** of $\overline{PP'}$. Points on ℓ do not move.

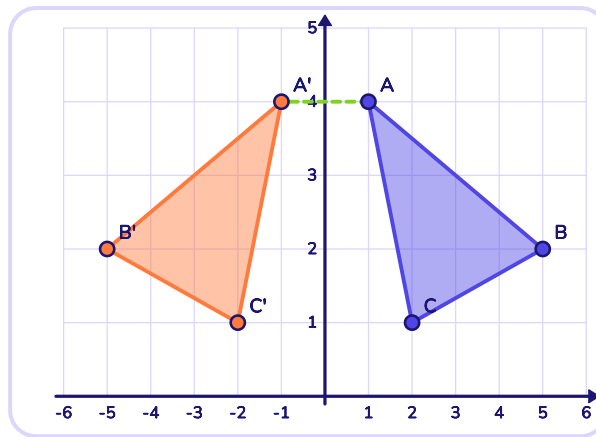
P Coordinate rules

In the x-axis: $(x, y) \rightarrow (x, -y)$. In the y-axis: $(x, y) \rightarrow (-x, y)$. In the line $y = x$: $(x, y) \rightarrow (y, x)$. In the line $y = -x$: $(x, y) \rightarrow (-y, -x)$.

E Example 2 : mirror across the y-axis

Reflect $A(1, 4)$, $B(5, 2)$, $C(2, 1)$ in the y-axis.

Change the sign of each x-coordinate: $A'(-1, 4)$, $B'(-5, 2)$, $C'(-2, 1)$. Each segment $\overline{AA'}$ is horizontal and is cut in half by the y-axis.



Careful

A reflection reverses the **orientation**: if you read A, B, C counterclockwise on the preimage, you read A', B', C' clockwise on the image. Translations and rotations keep the orientation.

4. Rotations



Rotation

A **rotation** about a point O (the **center**) by an angle θ turns every point around O by θ , keeping its distance to O . A **positive** angle means **counterclockwise**.



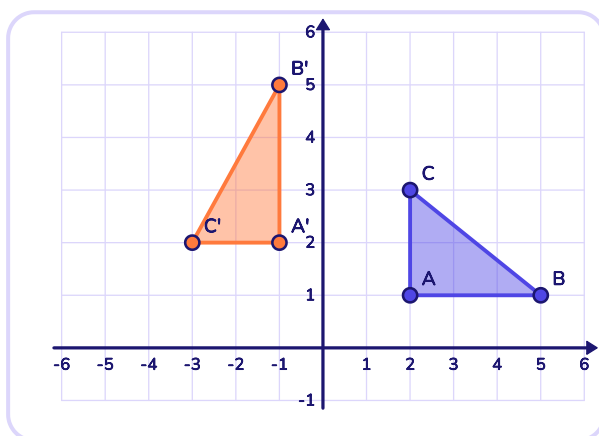
Coordinate rules about the origin

90° counterclockwise: $(x, y) \rightarrow (-y, x)$. 180° : $(x, y) \rightarrow (-x, -y)$. 270° counterclockwise (the same as 90° clockwise): $(x, y) \rightarrow (y, -x)$.

E Example 3 : a quarter turn

Rotate $A(2, 1)$, $B(5, 1)$, $C(2, 3)$ by 90° counterclockwise about the origin.

Apply $(x, y) \rightarrow (-y, x)$: $A'(-1, 2)$, $B'(-1, 5)$, $C'(-3, 2)$. The side $\overline{AB}$, horizontal and 3 units long, becomes the vertical side $\overline{A'B'}$, also 3 units long.



M Rotating about a point $C(h, k)$ other than the origin

1. Subtract the center: $(x - h, y - k)$.
2. Apply the origin rule for the angle.
3. Add the center back.

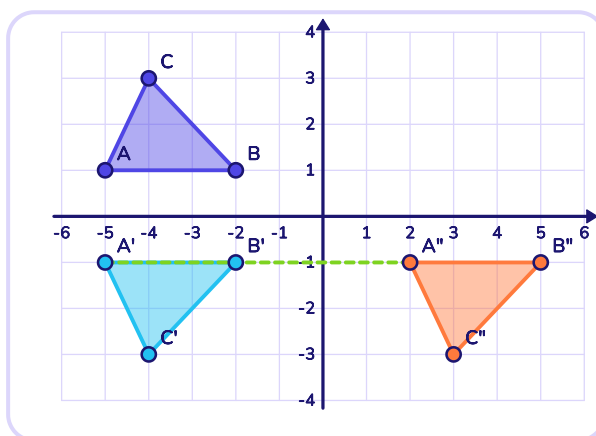
5. Compositions of transformations

A **composition** applies one transformation, then another to the result. **The order matters:** reflecting and then sliding usually does not give the same image as sliding and then reflecting.

E Example 4 : a glide reflection

Reflect $A(-5, 1)$, $B(-2, 1)$, $C(-4, 3)$ in the x-axis, then translate by $\langle 7, 0 \rangle$.

Reflection: $A'(-5, -1)$, $B'(-2, -1)$, $C'(-4, -3)$. Translation: $A''(2, -1)$, $B''(5, -1)$, $C''(3, -3)$.



P Two reflections

Reflecting in two **parallel** lines that are d units apart is a translation by $2d$, perpendicular to the lines. Reflecting in two lines that **intersect** at an angle θ is a rotation by 2θ about the intersection point.

! Zyro's tip

When you compose several moves, write the new coordinates after every step instead of trying to do everything in your head. One slip in the middle spoils the final answer.

6. Congruence through rigid motions

D Congruent figures

Two figures are **congruent** ($\cong$) if a rigid motion, or a sequence of rigid motions, maps one exactly onto the other. Then all **corresponding sides** and **corresponding angles** are congruent.

E Example 5 : checking with distances

Use the triangles of Example 1. $AC^2 = 1^2 + 2^2 = 5$ and $A'C'^2 = 1^2 + 2^2 = 5$. $BC^2 = 2^2 + 2^2 = 8$ and $B'C'^2 = 2^2 + 2^2 = 8$. Together with $AB = A'B' = 3$, the three pairs of sides match, so $\triangle ABC \cong \triangle A'B'C'$, as expected for a translation.

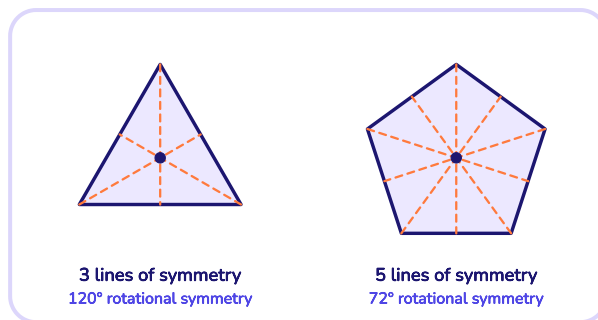
To prove two triangles congruent without drawing, show that they have three pairs of congruent sides (SSS), or use SAS or ASA. Each of these criteria guarantees that a sequence of rigid motions exists.

7. Symmetry of figures

D Symmetry

A figure has **line symmetry** if a reflection in some line maps it onto itself. It has **rotational symmetry** if a rotation of less than 360° about its center maps it onto itself.

A regular polygon with n sides has n lines of symmetry and rotational symmetry with smallest angle $\frac{360^\circ}{n}$. The equilateral triangle has 120° , the square 90° , the regular hexagon 60° .



8. Summary of coordinate rules

Transformation	Rule for (x, y)
Translation by $\langle a, b \rangle$	$(x + a, y + b)$
Reflection in the x-axis	$(x, -y)$
Reflection in the y-axis	$(-x, y)$
Reflection in $y = x$	(y, x)
Rotation 90° counterclockwise about the origin	$(-y, x)$
Rotation 180° about the origin	$(-x, -y)$
Rotation 270° counterclockwise about the origin	$(y, -x)$

Key takeaways

- A rigid motion keeps all distances and angle measures: preimage and image are congruent.
- Translation: $(x, y) \rightarrow (x + a, y + b)$. Reflection: the line of reflection is the perpendicular bisector of each $\overline{PP'}$.
- Rotation: positive angles turn counterclockwise; about the origin, 90° gives $(-y, x)$ and 180° gives $(-x, -y)$.
- The order of a composition matters. Two reflections give a translation (parallel lines) or a rotation (intersecting lines).
- Two figures are congruent when a sequence of rigid motions maps one onto the other.
- A regular n -gon has n lines of symmetry and a smallest rotation angle of $360^\circ \div n$.



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