

Systems of Linear Equations: math lesson, Grade 9 – download the PDF

Sometimes one equation is not enough to pin down a situation. Two unknown amounts, such as the price of a ticket and the price of a snack, need *two* pieces of information. A system of linear equations puts those pieces together, and solving it tells you the one pair of numbers that fits both at the same time. In this chapter you will learn three ways to solve a system, how to spot the special cases, and how to extend the ideas to inequalities.

1. What is a system of linear equations?

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System and solution

A **system of linear equations** is a set of two or more linear equations that use the same variables. A **solution** of the system is an ordered pair (x, y) that makes *every* equation true at the same time.

We usually write a system with a brace on the left, or we simply list the equations one under the other. A pair that satisfies only one of the equations is not a solution of the system: it has to work in all of them.

E**Example 1 : checking a pair**

Is $(4, 1)$ a solution of $x + y = 5$ and $2x - y = 7$?

First equation: $4 + 1 = 5$, true. Second equation: $2(4) - 1 = 7$, true. Both are true, so $(4, 1)$ is a solution of the system.

Now try $(3, 2)$. The first equation gives $3 + 2 = 5$, true, but the second gives $2(3) - 2 = 4 \neq 7$. So $(3, 2)$ is not a solution.

2. Solving by graphing

Each linear equation has a straight line as its graph, and every point of that line is a solution of that one equation. A point that lies on *both* lines is a solution of both equations. So the solution of the system is the point where the two lines cross.

M**Method : solving by graphing**

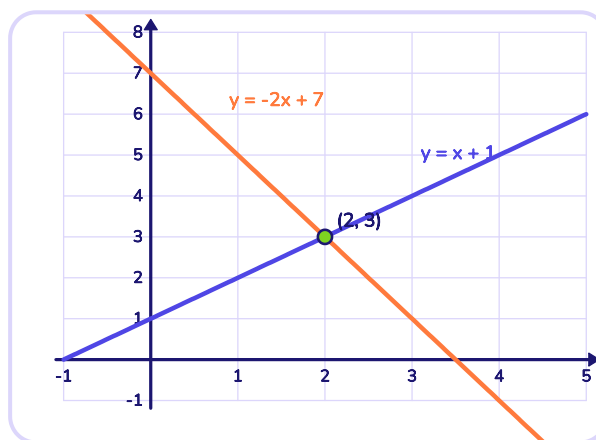
- Write each equation in slope-intercept form $y = mx + b$.
- Graph the first line: plot the y-intercept b , then use the slope m (rise over run) to find more points.
- Graph the second line on the same coordinate plane.
- Read the coordinates of the intersection point.
- Check the pair in both original equations.

E**Example 2 : two lines that cross**

Solve $y = x + 1$ and $y = -2x + 7$ by graphing.

The first line starts at $(0, 1)$ and goes up 1 for every 1 to the right. The second starts at $(0, 7)$ and goes down 2 for every 1 to the right. The lines cross at $(2, 3)$.

Check: $3 = 2 + 1$ and $3 = -2(2) + 7$. Both are true, so the solution is $(2, 3)$.

**Careful with graphing**

Graphing is only as accurate as your drawing. If the lines cross between grid marks, you can only estimate the answer. Always check the pair you read in the equations, and switch to an algebraic method when the intersection is not a clean point.

3. Solving by substitution

Substitution turns two equations into one. You solve one equation for a variable, then replace that variable in the other equation. The result has only one unknown, which you already know how to solve.

M**Method : substitution**

- Pick an equation and isolate one variable (choose a variable with coefficient 1 or -1 to avoid fractions).
- Substitute that expression into the *other* equation.
- Solve the new equation for the remaining variable.
- Put the value back into the isolated expression to find the second variable.
- Check in both equations.

E**Example 3 : substitution**

Solve $y = 2x - 9$ and $3x + 4y = 8$.

The first equation already gives y . Replace y in the second equation:

$3x + 4(2x - 9) = 8$. Distribute: $3x + 8x - 36 = 8$, so $11x = 44$ and $x = 4$.

Then $y = 2(4) - 9 = -1$. Check in the second equation:

$3(4) + 4(-1) = 12 - 4 = 8$. The solution is $(4, -1)$.

**Careful with parentheses**

When you substitute an expression with several terms, wrap it in parentheses.

Writing $3x + 4 \cdot 2x - 9$ instead of $3x + 4(2x - 9)$ forgets to multiply the -9 by 4.

4. Solving by elimination

Elimination adds or subtracts the equations so that one variable disappears. It works because you may add equal quantities to both sides of an equation. It is the favorite method when both

equations are in standard form $Ax + By = C$.

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Method : elimination

- Write both equations in standard form, with the variables lined up.
- Multiply one or both equations by a number so that one variable has opposite coefficients (for example $6y$ and $-6y$).
- Add the equations: that variable cancels. (If the coefficients are equal, subtract instead.)
- Solve for the remaining variable.
- Substitute back into one original equation to find the other variable, then check.

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Example 4 : elimination with multipliers

Solve $2x + 3y = 12$ and $5x - 2y = 11$.

To cancel y , make the coefficients $6y$ and $-6y$. Multiply the first equation by 2 and the second by 3: $4x + 6y = 24$ and $15x - 6y = 33$.

Add: $19x = 57$, so $x = 3$. Substitute in the first equation: $2(3) + 3y = 12$, so $3y = 6$ and $y = 2$.

Check in the second equation: $5(3) - 2(2) = 15 - 4 = 11$. The solution is $(3, 2)$.

5. No solution or infinitely many solutions

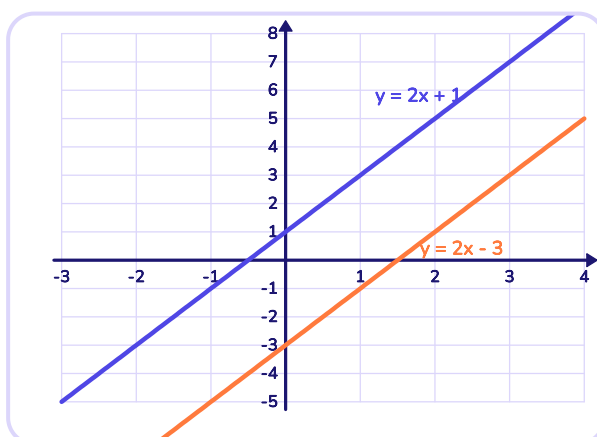
Two lines in a plane can cross at one point, never meet, or lie on top of each other. That gives three possible outcomes for a system.

Lines	Slopes and intercepts	Number of solutions	What the algebra shows
Cross	different slopes	exactly one	a single pair such as $x = 2$
Parallel	same slope, different intercepts	none	a false statement such as $0 = 5$
Same line	same slope, same intercept	infinitely many	a true statement such as $0 = 0$

E Example 5 : spotting the special cases

System (i): $y = 2x + 1$ and $4x - 2y = 6$. Solve the second equation for y :
 $-2y = -4x + 6$, so $y = 2x - 3$. Both lines have slope 2 but the intercepts 1 and -3 differ. The lines are parallel: **no solution**.

System (ii): $6x + 3y = 12$ and $y = -2x + 4$. From the first equation,
 $3y = -6x + 12$, so $y = -2x + 4$. It is the same line: **infinitely many solutions**.





Rule of thumb

If solving by substitution or elimination makes *all* variables vanish, look at what is left. A false statement ($0 = 5$) means no solution. A true statement ($0 = 0$) means infinitely many solutions.



6. Choosing a method

All three methods give the same answer, so pick the one that needs the least work.

Situation	Best method
Both equations are already graphed, or you need a quick picture	Graphing
One variable is already alone, such as $y = 3x - 5$	Substitution
Both equations are in standard form $Ax + By = C$	Elimination
A coefficient equals 1 or -1 in one equation	Substitution (or elimination)



Zyro's tip

On my home planet we say: look before you leap! Spend five seconds scanning the equations. A variable that is already isolated, or opposite coefficients that are already lined up, save you a whole step.

7. Word problems with systems

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Method : from words to a system

- Name the two unknowns with letters and say what each one means, including units.
- Translate each sentence of the problem into an equation (one equation per relationship).
- Solve the system with the method of your choice.
- Answer the question in a full sentence with units, and check against the story.

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Example 6 : prices

At a hiking shop, 3 water bottles and 2 snack bars cost 19 dollars. Two water bottles and 5 snack bars cost 20 dollars. Find each price.

Let b be the price of a bottle and s the price of a bar, in dollars. Then $3b + 2s = 19$ and $2b + 5s = 20$.

Multiply the first equation by 5 and the second by 2: $15b + 10s = 95$ and $4b + 10s = 40$. Subtract: $11b = 55$, so $b = 5$. Then $3(5) + 2s = 19$ gives $s = 2$.

A bottle costs 5 dollars and a bar costs 2 dollars. Check: $2(5) + 5(2) = 20$.

8. Systems of linear inequalities and solution regions

A **linear inequality** such as $y > x - 1$ has a whole half-plane of solutions. To graph it, draw the boundary line $y = x - 1$, then shade the side that makes the inequality true.

- Use a **dashed** line for $<$ or $>$ (points on the line are not solutions).

- Use a **solid** line for $\leq$ or $\geq$ (points on the line are solutions).
- To choose the side, test a point that is not on the line, such as $(0, 0)$.

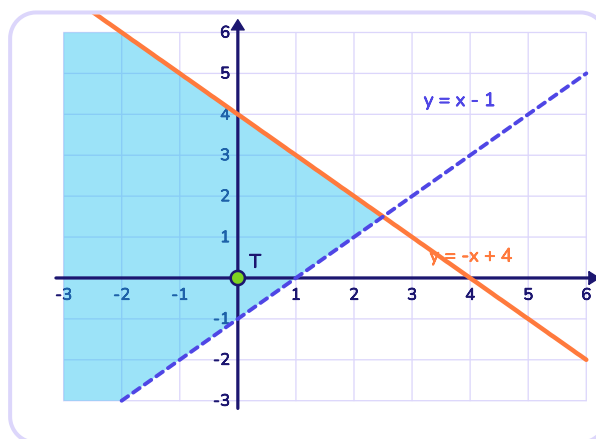
A **system of linear inequalities** asks for the points that satisfy all the inequalities together. Its graph is the **overlap** of the shaded half-planes, called the **solution region**.

E Example 7 : graphing a solution region

Graph $y > x - 1$ and $y \leq -x + 4$.

Draw the dashed line $y = x - 1$ and the solid line $y = -x + 4$. Test $(0, 0)$: $0 > -1$ is true and $0 \leq 4$ is true, so the region containing $(0, 0)$ is the solution region. The corner of the region is where the lines meet: $x - 1 = -x + 4$ gives $x = 2.5$ and $y = 1.5$.

The point $(3, 3)$ is not in the region: $3 > 2$ is true, but $3 \leq 1$ is false.



Careful with the boundary

A corner point where a dashed line is involved is not part of the solution region. Also, when you multiply or divide an inequality by a negative number, reverse the inequality sign.

Key takeaways

- A solution of a system is an ordered pair that makes every equation true.
- Graphing shows the solution as the intersection of the lines, but gives only approximate readings if the point is not on the grid.
- Substitution: isolate a variable, replace it in the other equation, solve, then back-substitute.
- Elimination: make opposite coefficients, add the equations, solve, then back-substitute.
- Different slopes give one solution; equal slopes with different intercepts give none; the same line gives infinitely many.
- For word problems, define the unknowns, write one equation per relationship, solve, and answer with units.
- The solution of a system of inequalities is the overlap of the half-planes: dashed boundary for $<$ and $>$, solid for $\leq$ and $\geq$.



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