

# Systems of Linear Equations: math lesson, Grade 8 – download the PDF

Two clues, two unknown numbers. That is the heart of a **system of linear equations**. In this chapter you will learn three ways to crack a system, graphing, substitution and elimination, and you will learn to tell in advance whether a system has one solution, none, or infinitely many.

## 1. What is a system of equations?

A single equation such as  $x + y = 7$  has many solutions:  $(1, 6)$ ,  $(2, 5)$ ,  $(3, 4)$  and so on. When you add a second condition, only some pairs survive. Two or more equations considered together form a **system**.

### D

#### Definition

A **system of two linear equations** in the variables  $x$  and  $y$  is a pair of equations such as

$$\begin{cases} y = 2x - 1 \\ x + y = 5 \end{cases}$$

A **solution** of the system is an ordered pair  $(x, y)$  that makes **both** equations true at the same time.

**E****Example 1: checking a solution**

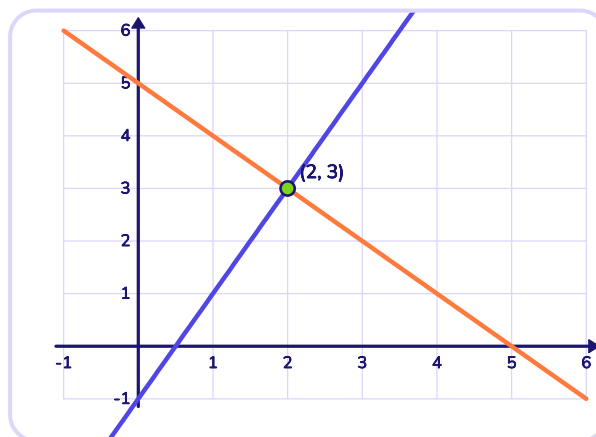
Is  $(2, 3)$  a solution of the system above? In the first equation,  $2(2) - 1 = 3$ , which equals  $y$ . In the second,  $2 + 3 = 5$ . Both are true, so  $(2, 3)$  is a solution. The pair  $(1, 4)$  fails:  $2(1) - 1 = 1 \neq 4$ .

**2. Solving by graphing**

Each linear equation graphs as a straight line, and every point on that line is a solution of that one equation. A point that lies on **both** lines satisfies both equations, so the solution of the system is the **point of intersection**.

**M****Method: graphing**

1. Rewrite each equation in slope-intercept form  $y = mx + b$ .
2. Graph the first line: plot the y-intercept  $b$ , then use the slope  $m$  (rise over run).
3. Graph the second line on the same coordinate plane.
4. Read the coordinates of the intersection point.
5. Check the pair in both original equations.



On the graph, the line  $y = 2x - 1$  has slope 2 and y-intercept  $-1$ ; the line  $y = -x + 5$  has slope  $-1$  and y-intercept 5. They meet at  $(2, 3)$ .



### Watch out

Graphing is only as accurate as your drawing. If the lines seem to cross between grid marks, you may read the point wrongly. Always check your answer in both equations, and switch to an algebraic method when the intersection is not clean.

## 3. Solving by substitution

In substitution you replace one variable by an expression taken from the other equation. This is the best method when one equation already says  $y = \dots$  or  $x = \dots$ , or when a variable has coefficient 1.



### Method: substitution

1. Isolate one variable in one equation.
2. Substitute that expression into the other equation. You now have one equation with one unknown.
3. Solve it.
4. Put the value back in the isolated equation to find the other variable.
5. Check in both original equations.

**E****Example 2: substitution**

Solve  $y = 3x - 4$  and  $2x + y = 11$ .

Replace  $y$  in the second equation:  $2x + (3x - 4) = 11$ , so  $5x - 4 = 11$ ,  $5x = 15$  and  $x = 3$ . Then  $y = 3(3) - 4 = 5$ . The solution is  $(3, 5)$ . Check:  $2(3) + 5 = 11$ . It works.

**!****Zyro's tip**

On my home planet we say: "Wrap the expression in parentheses before you substitute." Forgetting parentheses is the number one cause of sign mistakes, especially when you replace a variable by something like  $x - 4$  and there is a minus sign in front.

**4. Solving by elimination**

Elimination makes one variable disappear by adding or subtracting the equations. It is ideal when both equations are in standard form  $Ax + By = C$ .

**M****Method: elimination**

1. Line up the equations so that  $x$ ,  $y$  and the constants are in columns.
2. If needed, multiply one or both equations by a number so that one variable has opposite coefficients (like  $+3y$  and  $-3y$ ).
3. Add the equations. The variable with opposite coefficients disappears.
4. Solve for the remaining variable, then substitute back to find the other.
5. Check in both equations.

**E** Example 3: elimination with a multiplier

Solve  $2x + 3y = 13$  and  $5x - y = 7$ .

Multiply the second equation by 3:  $15x - 3y = 21$ . Add it to the first:  $17x = 34$ , so  $x = 2$ . Then  $5(2) - y = 7$  gives  $y = 3$ . The solution is  $(2, 3)$ . Check:  $2(2) + 3(3) = 13$ . It works.

Whatever you multiply, you must multiply **every term** on both sides of that equation, otherwise you change the problem.

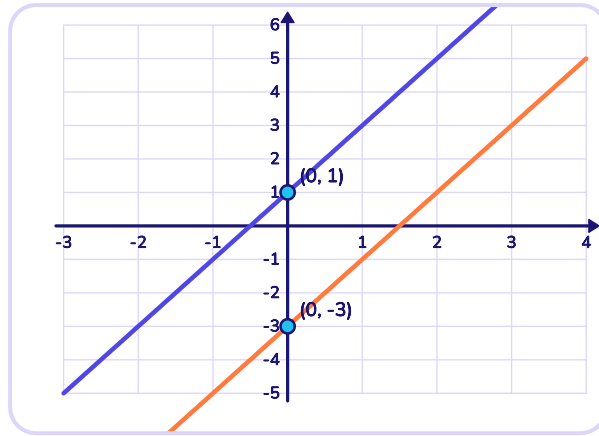
**5. How many solutions? Parallel and coincident lines**

Two lines in a plane can cross once, never cross, or be the very same line. Comparing slopes and y-intercepts tells you which case you are in without solving.

**P** Property

Two distinct lines with the same slope are parallel. Two lines with different slopes always intersect in exactly one point.

| Lines                  | Slopes    | Y-intercepts | Number of solutions |
|------------------------|-----------|--------------|---------------------|
| Intersecting           | different | any          | exactly one         |
| Parallel               | equal     | different    | none                |
| Coincident (same line) | equal     | equal        | infinitely many     |



#### **E** Example 4: no solution

The lines  $y = 2x + 1$  and  $y = 2x - 3$  both have slope 2, but they cross the y-axis at different points. They are parallel and never meet. Algebraically, substituting gives  $2x + 1 = 2x - 3$ , which simplifies to  $1 = -3$ : a false statement, so there is no solution.

By contrast,  $y = 2x + 1$  and  $4x - 2y = -2$  are the same line: solving the second for  $y$  gives  $y = 2x + 1$ . Substituting leads to a statement that is always true, such as  $0 = 0$ , so every point of the line is a solution.

## **6. Word problems with systems**

Many real situations have two unknown quantities and two facts. Translate the story into two equations, solve, and answer in a full sentence with units.

**M****Method: from words to a system**

1. Define your variables clearly (for example,  $a$  = number of adult tickets).
2. Write one equation for each fact in the story.
3. Solve with the most convenient method.
4. Check that your answers make sense (not negative, whole numbers when counting).

**E****Example 5: tickets**

A school play sold 40 tickets. Adult tickets cost 8 dollars and child tickets cost 5 dollars. The total was 257 dollars. How many of each?

Let  $a$  be adult tickets and  $c$  child tickets. Then  $a + c = 40$  and  $8a + 5c = 257$ . From the first,  $c = 40 - a$ . Substituting:  $8a + 5(40 - a) = 257$ , so  $3a + 200 = 257$ ,  $a = 19$  and  $c = 21$ . Check:  $8(19) + 5(21) = 152 + 105 = 257$ . The school sold 19 adult tickets and 21 child tickets.

**7. Which method should you choose?**

- **Graphing:** great for seeing the situation and for estimating, or when equations are already in slope-intercept form.
- **Substitution:** best when a variable is already isolated or has coefficient 1.
- **Elimination:** best when both equations are in standard form and coefficients line up or can be matched easily.

All three methods give the same answer. Pick the one that needs the fewest steps, and always verify in both equations.

## Key takeaways

- A solution of a system is an ordered pair that satisfies **both** equations.
- Graphically, the solution is the intersection point of the two lines.
- Substitution: isolate a variable, replace it in the other equation, solve, then back-substitute.
- Elimination: make coefficients opposite, add the equations, solve, then back-substitute.
- Different slopes give one solution; same slope with different intercepts gives none (parallel lines); same slope and same intercept gives infinitely many (coincident lines).
- In word problems, define variables, write one equation per fact, solve, check, and answer with units.



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