

Solving Quadratic Equations: math lesson, Grade 9 – download the PDF

A ball thrown in the air, a garden with a fixed area, a satellite dish with a curved shape: all of them lead to equations with an x^2 in them. In this chapter you will learn four ways to solve these equations, how to tell in advance how many solutions to expect, and how to pick the fastest method every time.

1. Quadratic equations in standard form

D Quadratic equation

A quadratic equation in one variable can be written in **standard form**

$ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$. A **solution** (or **root**) is a value of x that makes the equation true.

To use any method below, first move every term to one side so the other side is **0**. For example, $3x^2 = 2x + 7$ becomes $3x^2 - 2x - 7 = 0$, so $a = 3, b = -2, c = -7$.

You already know one method: **factoring** with the zero-product property. If $A \cdot B = 0$, then $A = 0$ or $B = 0$. For instance $x^2 - 5x + 6 = (x - 2)(x - 3) = 0$ gives $x = 2$ or $x = 3$. The trouble is that many quadratics do not factor nicely, which is why we need the other tools.

2. The square root method

P Square root property

If $k \geq 0$, then $x^2 = k$ has the solutions $x = \sqrt{k}$ and $x = -\sqrt{k}$, written $x = \pm\sqrt{k}$. If $k < 0$ there is no real solution, because a square is never negative. In the same way, $(x - h)^2 = k$ gives $x = h \pm \sqrt{k}$.

E Example 1

Solve $3x^2 - 75 = 0$. Isolate the square: $3x^2 = 75$, so $x^2 = 25$ and $x = \pm 5$. Check: $3 \cdot 25 - 75 = 0$. The solutions are 5 and -5 .

E Example 2

Solve $(x - 2)^2 = 18$. Then $x - 2 = \pm\sqrt{18} = \pm 3\sqrt{2}$, so $x = 2 \pm 3\sqrt{2}$. Rounded to the nearest hundredth, $x \approx 6.24$ or $x \approx -2.24$.

⚠ Do not lose the second solution

Writing only $x = 5$ for $x^2 = 25$ forgets $x = -5$. Always write the $\pm$ sign when you take a square root.

3. Completing the square

When the equation has an x term, you can rewrite it as a perfect square so that the square root method works. The key fact is $(x + p)^2 = x^2 + 2px + p^2$: to complete $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.

M**Method: completing the square**

1. Divide by a so the coefficient of x^2 is 1.
2. Move the constant to the right side: $x^2 + bx = -c$.
3. Add $\left(\frac{b}{2}\right)^2$ to **both** sides.
4. Write the left side as a square: $\left(x + \frac{b}{2}\right)^2 = \text{number}$.
5. Use the square root method, then solve for x .

E**Example 3**

Solve $x^2 + 6x - 7 = 0$. Write $x^2 + 6x = 7$. Half of 6 is 3, and $3^2 = 9$, so add 9 to both sides: $x^2 + 6x + 9 = 16$, that is $(x + 3)^2 = 16$. Then $x + 3 = \pm 4$, so $x = 1$ or $x = -7$. Check $x = -7$: $49 - 42 - 7 = 0$.

E**Example 4**

Solve $2x^2 - 8x + 3 = 0$. Divide by 2: $x^2 - 4x + \frac{3}{2} = 0$, so $x^2 - 4x = -\frac{3}{2}$. Add 4: $(x - 2)^2 = \frac{5}{2}$. Then $x = 2 \pm \sqrt{\frac{5}{2}} = \frac{4 \pm \sqrt{10}}{2}$, about 3.58 or 0.42.

4. The quadratic formula

If you complete the square on the general equation $ax^2 + bx + c = 0$, you obtain a formula that solves *every* quadratic equation.

P**Quadratic formula**

The solutions of $ax^2 + bx + c = 0$ (with $a \neq 0$) are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

E**Example 5**

Solve $2x^2 + 5x - 3 = 0$. Here $a = 2$, $b = 5$, $c = -3$. Then

$$b^2 - 4ac = 25 + 24 = 49, \text{ so } x = \frac{-5 \pm 7}{4}. \text{ That gives } x = \frac{2}{4} = \frac{1}{2} \text{ or } x = \frac{-12}{4} = -3.$$

A**Watch the signs**

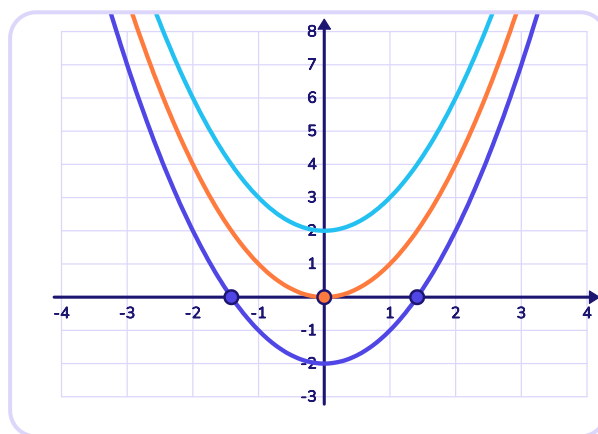
When b is negative, $-b$ is positive. For $x^2 - 4x - 1 = 0$, $b = -4$ so $-b = 4$ and

$$b^2 = (-4)^2 = 16. \text{ The solutions are } x = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}.$$

5. The discriminant**D****Discriminant**

The expression under the square root, $\Delta = b^2 - 4ac$, is called the **discriminant**. Its sign tells you how many real solutions the equation has before you solve it.

Value of Δ	Real solutions	Graph of $y = ax^2 + bx + c$
$\Delta > 0$	two different solutions	crosses the x-axis twice
$\Delta = 0$	exactly one solution, $x = -\frac{b}{2a}$	touches the x-axis at its vertex
$\Delta < 0$	no real solution	never reaches the x-axis



E Example 6

For $x^2 - 6x + 9 = 0$: $\Delta = 36 - 36 = 0$, one solution $x = 3$. For $x^2 + 2x + 5 = 0$: $\Delta = 4 - 20 = -16 < 0$, no real solution. For $3x^2 - x - 2 = 0$: $\Delta = 1 + 24 = 25 > 0$, two solutions $x = \frac{1 \pm 5}{6}$, namely 1 and $-\frac{2}{3}$.

When Δ is a perfect square (1, 4, 9, 16, 25, ...) and a, b, c are integers, the solutions are rational and the equation can be factored.

6. Choosing a solving method

Every method gives the same answers, so choose the one that costs the least work.

Shape of the equation	Best first choice
$ax^2 = k$ or $(x - h)^2 = k$ (no x term, or already a square)	square root method
$x^2 + bx + c = 0$ with easy integer factors	factoring and the zero-product property
$a = 1$ and b even, answer wanted as $h \pm \sqrt{k}$	completing the square
anything else, especially with messy numbers	quadratic formula
a picture or a rough answer is enough	graphing



Tip from Zyro

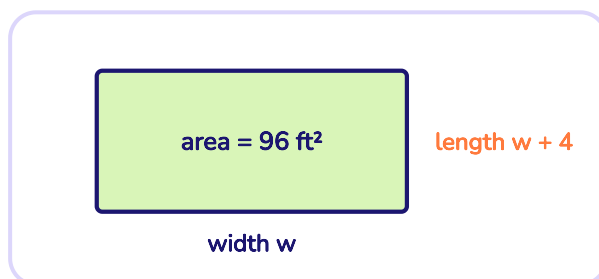
Always compute Δ first. If it is negative you can stop at once, and if it is a perfect square the problem will have clean answers.

7. Quadratic word problems

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Method: word problems

1. Name the unknown and say what it measures, with its unit.
2. Write an equation from the story, then put it in standard form.
3. Solve it with a suitable method.
4. Reject solutions that make no sense (a negative length or time), then answer in a sentence.



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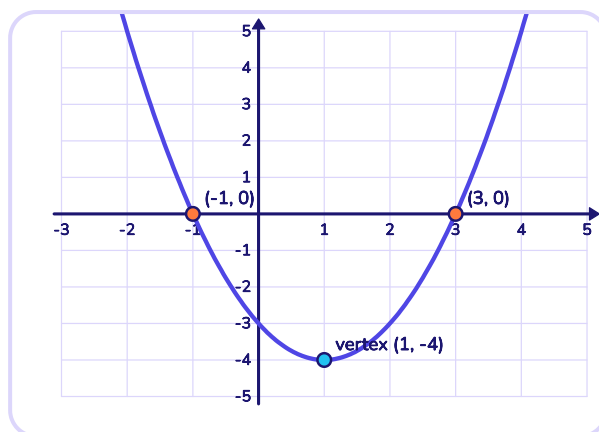
Example 7

A garden is 4 feet longer than it is wide, and its area is 96 square feet. Let w be the width in feet. Then $w(w + 4) = 96$, so $w^2 + 4w - 96 = 0$. Here

$\Delta = 16 + 384 = 400$, so $w = \frac{-4 \pm 20}{2}$, which gives $w = 8$ or $w = -12$. A width cannot be negative, so $w = 8$ ft and the length is 12 ft. Check: $8 \times 12 = 96$.

8. Solving by graphing

The solutions of $ax^2 + bx + c = 0$ are the **x-coordinates of the points where the parabola $y = ax^2 + bx + c$ meets the x-axis**. These points are called the *x-intercepts* or zeros.



On the graph of $y = x^2 - 2x - 3$ the parabola crosses the x -axis at $x = -1$ and $x = 3$, so those are the solutions of $x^2 - 2x - 3 = 0$. Graphing is quick, but when the intercepts are not whole numbers you can only *estimate* them (for example, between 4 and 5), so use a formula for an exact value. To solve $x^2 - 2x - 3 = k$ instead, look where the parabola meets the horizontal line $y = k$.

9. Linear-quadratic systems

A **linear-quadratic system** pairs a line with a parabola. A line and a parabola can meet in **two** points, one point (tangent), or no point.

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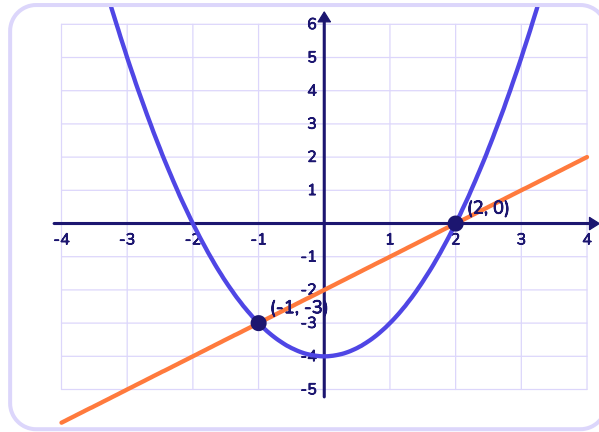
Method: substitution

1. Set the two expressions for y equal to each other.
2. Move everything to one side and solve the quadratic equation for x .
3. Substitute each x into the *linear* equation to find y .

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Example 8

Solve $y = x^2 - 4$ and $y = x - 2$. Then $x^2 - 4 = x - 2$, so $x^2 - x - 2 = 0$, that is $(x - 2)(x + 1) = 0$ and $x = 2$ or $x = -1$. With $y = x - 2$: the points are $(2, 0)$ and $(-1, -3)$.



Key takeaways

- Put the equation in standard form $ax^2 + bx + c = 0$ before choosing a method.
- Square root method: $x^2 = k$ gives $x = \pm\sqrt{k}$; never forget the negative root.
- Completing the square adds $\left(\frac{b}{2}\right)^2$ to both sides once $a = 1$.
- Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ works for every quadratic equation.
- $\Delta = b^2 - 4ac$: positive means two solutions, zero means one, negative means none.
- In word problems, check each solution against the story before answering.
- Solutions are the x-intercepts of the parabola; a line and a parabola meet in 0, 1 or 2 points.



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