

Solving Linear Equations: math lesson, Grade 9 – download the PDF

An equation is a balanced statement, and solving it means finding the value that keeps the balance true. In this chapter you will build a complete toolkit: you will read and write algebraic expressions, respect the order of operations, use the properties of equality, and solve equations step by step, from the simplest one-step puzzle up to equations with variables on both sides. You will even rearrange formulas and spot equations that have no solution or endlessly many.

1. Variables and expressions

A **variable** is a letter that stands for a number that can change or is not known yet. An **algebraic expression** combines numbers, variables and operations, but it has no equal sign. For example, $3n + 5$ is an expression: if $n = 4$, its value is $3 \cdot 4 + 5 = 17$.

D Vocabulary

In $7x - 2y + 9$, the **terms** are $7x$, $-2y$ and 9 . The number multiplying a variable is its **coefficient** (here 7 and -2), and a term with no variable (here 9) is a **constant**. **Like terms** have exactly the same variable part, such as $4x$ and $9x$, and they can be combined: $4x + 9x = 13x$.

To **evaluate** an expression, replace each variable by its value (use parentheses around negative values) and then calculate. To translate words into algebra, look for key phrases: “more than” and

“sum” mean addition, “less than” and “decreased by” mean subtraction, “product” and “times” mean multiplication, and “quotient” means division.

E

Example 1: evaluate and translate

Evaluate $2x^2 - 5x$ for $x = -3$: $2(-3)^2 - 5(-3) = 2 \cdot 9 + 15 = 33$.

Translate “6 less than four times a number n ”: $4n - 6$. Careful: the order is reversed, because “6 less than” means you subtract 6 from the other quantity.

2. Order of operations

Everyone must get the same answer from the same calculation, so mathematicians agreed on an order (often remembered as PEMDAS):

1. **P**arentheses (and other grouping symbols, including fraction bars);
2. **E**xponents;
3. **M**ultiplication and **D**ivision, from left to right;
4. **A**ddition and **S**ubtraction, from left to right.



Common mistake

Multiplication does not always come before division, and addition does not always come before subtraction. They share the same level, so you work from left to right:

$$24 \div 4 \cdot 3 = 6 \cdot 3 = 18, \text{ not } 24 \div 12 = 2.$$

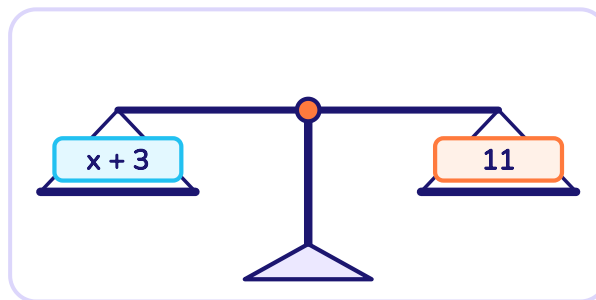
E**Example 2: a calculation with every level**

$$30 - 4(2 + 3)^2 \div 5 = 30 - 4 \cdot 5^2 \div 5 = 30 - 4 \cdot 25 \div 5 = 30 - 100 \div 5 = 30 - 20 =$$

The same order is used inside equations. When you check a solution, you substitute it and follow PEMDAS on each side.

3. Properties of equality

An equation says that two expressions have the same value. Think of a balance scale: if the two pans are level and you treat both pans in exactly the same way, they stay level.



P**Properties of equality**

If $a = b$, then for any number c :

- Addition: $a + c = b + c$;
- Subtraction: $a - c = b - c$;
- Multiplication: $a \cdot c = b \cdot c$;
- Division: $\frac{a}{c} = \frac{b}{c}$, as long as $c \neq 0$.

Also, the **Substitution Property** lets you replace a quantity by an equal quantity, and the **Distributive Property** says $a(b + c) = ab + ac$.

The goal is always to **isolate the variable**: get it alone on one side. To do that, you undo each operation using its **inverse operation**: addition and subtraction undo each other, and so do multiplication and division.

4. One-step and two-step equations**M****Method: solve an equation**

1. Find the operation applied to the variable.
2. Do the inverse operation on **both** sides.
3. Simplify and write the solution.
4. Check by substituting the value in the original equation.

E**Example 3: a one-step equation**

Solve $x + 3 = 11$. Subtract 3 on both sides: $x + 3 - 3 = 11 - 3$, so $x = 8$. Check: $8 + 3 = 11$. True.

A **two-step equation** such as $ax + b = c$ has two operations on the variable. Undo them in reverse order: first the addition or subtraction, then the multiplication or division.

E**Example 4: a two-step equation**

Solve $\frac{x}{4} - 7 = 2$. Add 7: $\frac{x}{4} = 9$. Multiply by 4: $x = 36$. Check: $\frac{36}{4} - 7 = 9 - 7 = 2$. True.

!**Zyro's tip**

On my home planet we say: "Wrap the present, then tie the ribbon; to open it, remove the ribbon first." Undoing the operations happens in the opposite order of the order of operations.

5. Multi-step equations

When an equation has parentheses or several like terms, simplify each side first. Distribute, combine like terms, and only then use the properties of equality.

E**Example 5: distribute, combine, solve**

Solve $3(2x - 1) + 4x = 47$.

Distribute: $6x - 3 + 4x = 47$. Combine like terms: $10x - 3 = 47$. Add 3: $10x = 50$.

Divide by 10: $x = 5$. Check: $3(10 - 1) + 20 = 27 + 20 = 47$. True.

When an equation contains fractions, you can multiply every term by the least common denominator to clear them. For $\frac{x}{2} + \frac{x}{3} = 10$, multiplying by 6 gives $3x + 2x = 60$, so $x = 12$. For decimals, multiply by a power of 10 in the same way.

**Common mistake**

When you distribute, multiply **every** term inside the parentheses:

$3(x - 4) = 3x - 12$, not $3x - 4$. Watch the signs when the factor is negative:

$-2(x - 5) = -2x + 10$.

6. Variables on both sides

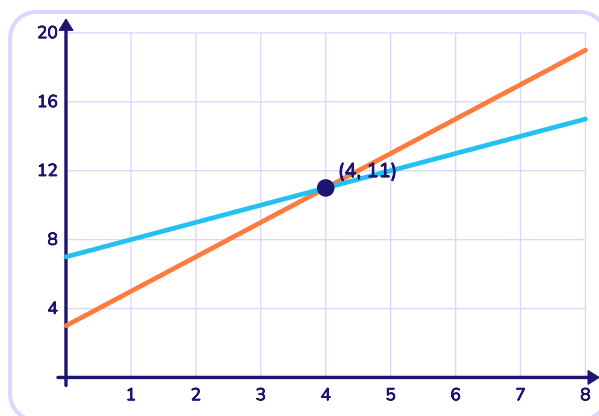
If the variable appears on both sides, move all variable terms to one side and all constants to the other. Then you are back to a two-step equation.

E**Example 6: variables on both sides**

Solve $2x + 3 = x + 7$. Subtract x from both sides: $x + 3 = 7$. Subtract 3: $x = 4$.

Check: left side $2 \cdot 4 + 3 = 11$, right side $4 + 7 = 11$.

There is a nice picture of this. Each side of the equation is a linear function, and the solution is the x -coordinate of the point where the two lines meet.



The lines $y = 2x + 3$ and $y = x + 7$ cross at $(4, 11)$: the common value of both sides is 11, and the solution of the equation is $x = 4$.

7. Literal equations

A **literal equation** is an equation with several letters, such as a formula. To “solve for” one letter, treat the other letters like numbers and isolate the chosen variable with the same properties of equality.

E Example 7: rearranging a formula

The perimeter of a rectangle is $P = 2l + 2w$. Solve for w : subtract $2l$ to get

$P - 2l = 2w$, then divide by 2: $w = \frac{P - 2l}{2}$. With $P = 40$ and $l = 12$:

$$w = \frac{40 - 24}{2} = 8.$$

Another classic is the temperature formula $F = 1.8C + 32$ (Fahrenheit and Celsius). Solving for C gives $C = \frac{F - 32}{1.8}$. So 68°F corresponds to $\frac{68 - 32}{1.8} = 20^\circ\text{C}$.

When you divide by an expression containing a letter, such as m in $y = mx + b$, remember that this letter must not be zero.

8. Equations with no solution or infinitely many solutions

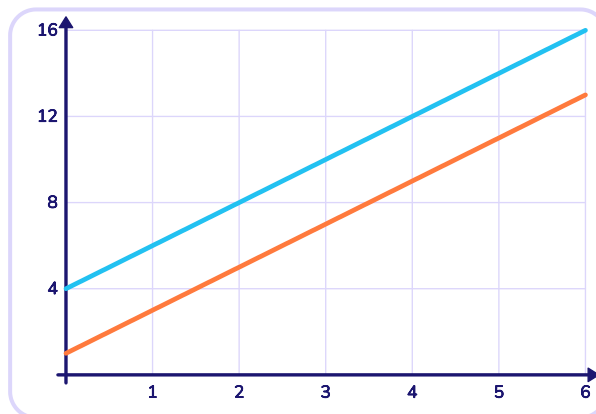
Most linear equations have exactly one solution. But simplifying can lead to two other outcomes.

P**Three possible outcomes**

- You reach $x =$ a number: **one solution**.
- You reach a false statement such as $3 = 8$: **no solution**, because no value of x can ever work.
- You reach a statement that is always true such as $5 = 5$: **infinitely many solutions**, because every real number works (the two sides are identical).

E**Example 8: no solution**

Solve $2x + 1 = 2x + 4$. Subtract $2x$ from both sides: $1 = 4$. This is false, so there is no solution. The graphs agree: the lines $y = 2x + 1$ and $y = 2x + 4$ have the same slope and never meet.

**E****Example 9: infinitely many solutions**

Solve $4(x + 2) = 4x + 8$. Distribute: $4x + 8 = 4x + 8$. Subtract $4x$: $8 = 8$. This is always true, so every real number is a solution.



Common mistake

Do not write $x = 0$ when you reach $0 = 0$, and do not write “ $x = \text{nothing}$ ” when you reach $1 = 4$. Say clearly “all real numbers” or “no solution”.

Key takeaways

- An expression has no equal sign; an equation states that two expressions are equal.
- Follow PEMDAS: parentheses, exponents, then multiplication and division, then addition and subtraction, both from left to right.
- Whatever you do to one side of an equation, do the same to the other side.
- Undo operations with inverse operations, in reverse order, to isolate the variable; always check your answer.
- Simplify first: distribute and combine like terms; move variable terms to one side when they appear on both sides.
- To solve a literal equation for a letter, isolate that letter just as you would with a number.
- A false statement means no solution; a statement that is always true means infinitely many solutions.



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