

Slope and Graphing Linear Functions: math lesson, Grade 9 – download the PDF

A hiker climbing a trail, a taxi meter ticking upward, a candle slowly shrinking: all of these change at a steady pace, and a straight line on a graph captures that pace in a single number called the slope. In this chapter you will learn to measure slope, to read it from an equation, and to write the equation of a line in three different forms. You will also graph lines quickly, recognize parallel and perpendicular lines, and spot direct variation.

1. Rate of change

A **rate of change** compares how much one quantity changes with how much another quantity changes. Speed is a rate of change (miles per hour), and so is the price of gasoline (dollars per gallon). Look at this bike rental shop.

Hours rented, x	Total cost in dollars, y
2	12
5	27
8	42

From 2 hours to 5 hours the cost grows by $27 - 12 = 15$ dollars while the time grows by $5 - 2 = 3$ hours, so the rate is $\frac{15}{3} = 5$ dollars per hour. From 5 hours to 8 hours we get

$$\frac{42 - 27}{8 - 5} = \frac{15}{3} = 5 \text{ again. The rate never changes.}$$

D Linear function

A function is **linear** when its rate of change is the same between any two points of its table or graph. The graph is then a straight line, and the constant rate of change is called the **slope**.

E Example 1: a constant rate

A tank holds 40 gallons after 2 minutes of filling and 70 gallons after 5 minutes. The rate of change is $\frac{70 - 40}{5 - 2} = \frac{30}{3} = 10$ gallons per minute (about 37.9 liters per minute).

2. The slope formula

Pick two points on a line, (x_1, y_1) and (x_2, y_2) . Going from the first to the second, the **rise** is the vertical change and the **run** is the horizontal change.

D Slope

The slope m of a line is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2.$$



In the figure, going from $A(1, 1)$ to $B(5, 4)$ the run is $5 - 1 = 4$ and the rise is $4 - 1 = 3$, so $m = \frac{3}{4}$.

M Method: find a slope from two points

1. Label the points (x_1, y_1) and (x_2, y_2) .
2. Subtract the y -values: $y_2 - y_1$.
3. Subtract the x -values in the same order: $x_2 - x_1$.
4. Divide and simplify.

E Example 2: a negative slope

For $(-2, 5)$ and $(4, -1)$: $m = \frac{-1 - 5}{4 - (-2)} = \frac{-6}{6} = -1$. The line goes down one unit for every unit to the right.

The sign and the value of the slope tell you how the line looks.

Type of line	Slope	What you see
Rising	positive	goes up from left to right
Falling	negative	goes down from left to right
Horizontal	0	flat, y never changes
Vertical	undefined	straight up and down, x never changes

E Example 3: zero and undefined slopes

Through $(-5, 2)$ and $(6, 2)$: $m = \frac{2 - 2}{6 - (-5)} = \frac{0}{11} = 0$. Through $(3, -4)$ and $(3, 7)$: the run is $3 - 3 = 0$, and dividing by zero is impossible, so the slope is undefined.

A Watch the order

Subtract in the same order on top and on the bottom. Computing $\frac{y_2 - y_1}{x_1 - x_2}$ flips the sign of the slope. Also be careful with negative numbers: $4 - (-2) = 6$, not 2.

3. Slope-intercept form

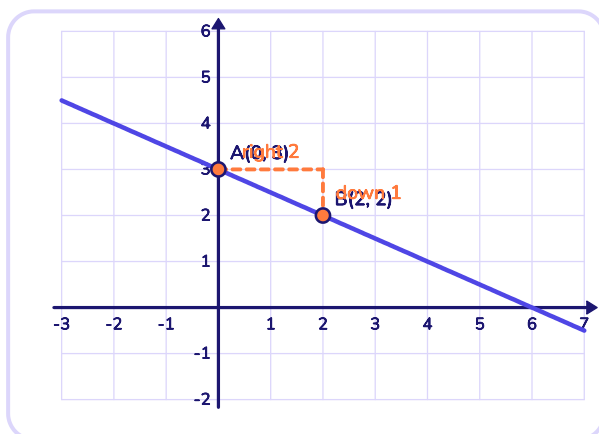
D Slope-intercept form

A line with slope m that crosses the y -axis at $(0, b)$ has the equation

$$y = mx + b.$$

The number b is the y -intercept.

This form is the fastest way to graph: plot the y -intercept first, then use the slope as a rise over run to find more points.



E Example 4: graph a line

Graph $y = -\frac{1}{2}x + 3$. The y -intercept is **3**, so plot **A(0, 3)**. The slope $-\frac{1}{2} = \frac{-1}{2}$ means: go right 2, down 1, which gives **B(2, 2)**. Draw the line through the two points.

E Example 5: write an equation

A line passes through $(0, 5)$ and $(2, 1)$. The point $(0, 5)$ is the y -intercept, so $b = 5$.

The slope is $m = \frac{1 - 5}{2 - 0} = -2$. The equation is $y = -2x + 5$.

! Zyro's tip

On my home planet we always test a new equation with a point we already know. Put its coordinates in the equation: if both sides match, the line really goes through that point.

4. Point-slope form

Sometimes you know the slope and a point that is *not* on the y -axis. Point-slope form handles this case directly.

P Point-slope form

The line with slope m through the point (x_1, y_1) has the equation

$$y - y_1 = m(x - x_1).$$

E Example 6: slope and a point

Slope 4, point $(2, -3)$: $y - (-3) = 4(x - 2)$, so $y + 3 = 4x - 8$ and $y = 4x - 11$.

E**Example 7: two points**

Through $(1, 7)$ and $(4, 1)$: the slope is $m = \frac{1-7}{4-1} = -2$. Using the first point, $y - 7 = -2(x - 1)$, so $y = -2x + 9$. Check with the second point: $-2 \cdot 4 + 9 = 1$. It works.

**Sign trap**

If y_1 or x_1 is negative, the minus sign in the formula becomes a plus: for $(2, -3)$ we write $y + 3$, not $y - 3$.

5. Standard form**D****Standard form**

An equation of the form $Ax + By = C$, where A , B and C are integers and A is not negative, is written in **standard form**. When $B \neq 0$, the slope is $-\frac{A}{B}$ and the y -intercept is $\frac{C}{B}$.

E**Example 8: convert to standard form**

Start with $y = -\frac{2}{3}x + 4$. Multiply every term by 3 to clear the fraction:

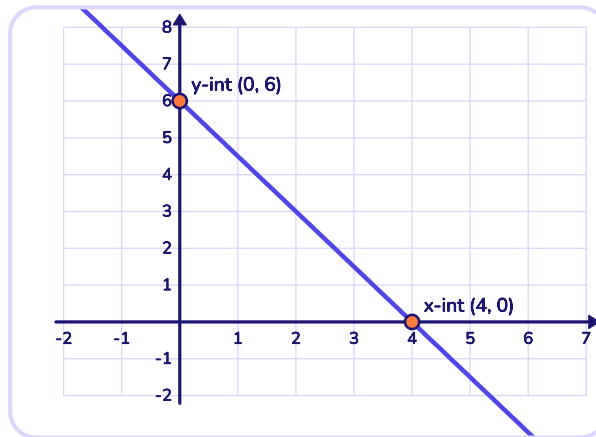
$3y = -2x + 12$. Move the x -term to the left: $2x + 3y = 12$. The slope is $-\frac{2}{3}$, as expected.

6. Graphing with intercepts

The **x -intercept** is where the line crosses the x -axis (there $y = 0$); the **y -intercept** is where it crosses the y -axis (there $x = 0$). Standard form makes both easy to find.

M Method: graph with intercepts

1. Replace y by 0 and solve for x : this gives the x -intercept.
2. Replace x by 0 and solve for y : this gives the y -intercept.
3. Plot both points and draw the line through them.



For $3x + 2y = 12$: with $y = 0$, $3x = 12$ and $x = 4$; with $x = 0$, $2y = 12$ and $y = 6$. The line goes through $(4, 0)$ and $(0, 6)$.

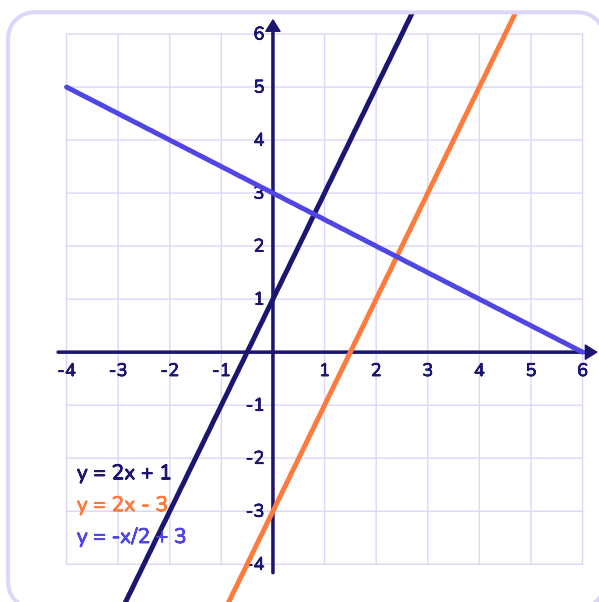
E Example 9: a negative intercept

For $5x - 4y = 20$: if $y = 0$ then $x = 4$; if $x = 0$ then $-4y = 20$, so $y = -5$. The intercepts are $(4, 0)$ and $(0, -5)$, and the slope is $-\frac{5}{-4} = \frac{5}{4}$.

7. Parallel and perpendicular lines

P Slopes of special pairs

- Two different non-vertical lines are **parallel** exactly when they have the same slope.
- Two non-vertical lines are **perpendicular** exactly when the product of their slopes is -1 : the slopes are opposite reciprocals, such as 2 and $-\frac{1}{2}$, or $\frac{3}{4}$ and $-\frac{4}{3}$.



E Example 10: perpendicular through a point

Find the line perpendicular to $y = 2x + 1$ that passes through $(4, 1)$. The new slope is $-\frac{1}{2}$. Point-slope form gives $y - 1 = -\frac{1}{2}(x - 4)$, so $y = -\frac{1}{2}x + 3$. A parallel line to $y = 2x + 1$ through $(1, -1)$ would be $y + 1 = 2(x - 1)$, that is $y = 2x - 3$.

8. Direct variation

D Direct variation

We say that **y varies directly** with **x** when **$y = kx$** for a constant **$k \neq 0$** , called the **constant of variation**. Its graph is a line through the origin **$(0, 0)$** , and **$k = \frac{y}{x}$** is also its slope.

E Example 11: trail mix

Three pounds of trail mix (about 1.36 kilograms) cost 14.40 dollars. The constant is $k = \frac{14.40}{3} = 4.8$ dollars per pound, so $c = 4.8p$. For 7.5 pounds:
 $c = 4.8 \times 7.5 = 36$ dollars.

E Example 12: is it direct variation?

The points **$(2, 5)$** and **$(4, 9)$** give $\frac{5}{2} = 2.5$ and $\frac{9}{4} = 2.25$. The ratios differ, so **y** does not vary directly with **x** .

⚠ Not every line is direct variation

$y = 3x + 2$ is linear, but it does not pass through the origin, so it is not direct variation. Only equations of the form **$y = kx$** are.

Key takeaways

- The slope is the constant rate of change: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- Positive slope rises, negative slope falls, horizontal lines have slope **0**, vertical lines have undefined slope.
- Slope-intercept form: $y = mx + b$; point-slope form: $y - y_1 = m(x - x_1)$; standard form: $Ax + By = C$.
- To graph with intercepts, set $y = 0$ and then $x = 0$.
- Parallel lines have equal slopes; perpendicular lines have slopes whose product is -1 .
- Direct variation: $y = kx$, a line through the origin with $k = \frac{y}{x}$.



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