

Similarity and Dilations: math lesson, Grade 10 – download the PDF

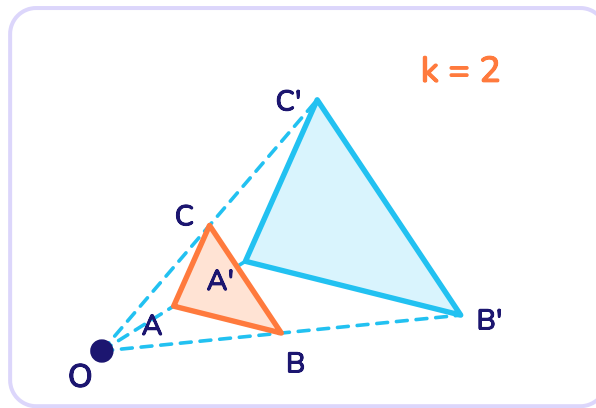
A photo on your phone, a floor plan, a model rocket: all of them keep the *shape* of the real thing while changing its *size*. In this chapter you will learn the geometry behind that idea. You will use dilations to build bigger and smaller copies, prove that two triangles are similar, and then use similarity to find lengths, perimeters, and areas that you cannot measure directly.

1. Dilations and the scale factor

A **dilation** stretches or shrinks a figure from a fixed point. Every point of the figure moves along a ray that starts at that point, called the **center**, and its distance from the center is multiplied by the same number.

D Definition: dilation

A dilation with center O and scale factor $k > 0$ sends each point P to the point P' on ray OP such that $OP' = k \cdot OP$. If $k > 1$ the dilation is an **enlargement**. If $0 < k < 1$ it is a **reduction**. If $k = 1$ nothing moves.



Notice what does *not* change: every angle keeps its measure, and every line is sent to a parallel line (a line through the center stays on itself). What does change is length: each segment is multiplied by k .

P Dilations in the coordinate plane

A dilation centered at the origin with scale factor k follows the rule $(x, y) \mapsto (kx, ky)$. For a center $C(h, v)$, the rule becomes $(x, y) \mapsto (h + k(x - h), v + k(y - v))$.

E Example 1: dilating a triangle

Dilate $A(2, 3)$, $B(6, 3)$, and $C(2, 7)$ from the origin with $k = 1.5$.

$A' = (3, 4.5)$, $B' = (9, 4.5)$, $C' = (3, 10.5)$. Side AB has length 4 and $A'B'$ has length 6, and $6 \div 4 = 1.5 = k$. The image is a bigger copy of the same shape.

2. Similar figures and proportions

Two figures are **similar** when one can be turned into the other by a dilation followed by moves that keep size and shape (slides, turns, flips). We write $\triangle ABC \sim \triangle DEF$.

D Definition: similar polygons

Two polygons are similar if (1) all pairs of corresponding angles are congruent and (2) all pairs of corresponding sides are proportional. The common ratio of the sides is the **scale factor**.

The order of the letters matters: $\triangle ABC \sim \triangle DEF$ means **A** corresponds to **D**, **B** to **E**, and **C** to **F**, so

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC} = k.$$

E Example 2: finding missing sides

Suppose $\triangle ABC \sim \triangle DEF$ with $AB = 6$, $BC = 9$, $AC = 12$, and $DE = 8$. Then $k = \frac{8}{6} = \frac{4}{3}$, so $EF = 9 \cdot \frac{4}{3} = 12$ and $DF = 12 \cdot \frac{4}{3} = 16$.



Careful

Equal angles alone are not enough for polygons with more than three sides. A 2 by 6 rectangle and a 3 by 8 rectangle have four right angles each, yet $\frac{3}{2} \neq \frac{8}{6}$, so they are not similar. Triangles are special: they need less information, as the next part shows.

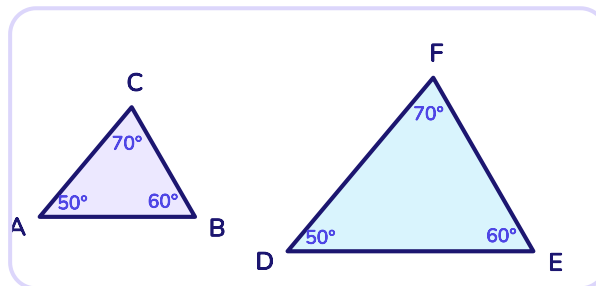
3. AA, SSS, and SAS similarity

You do not need to check all three angles and all three sides. Each of the following shortcuts is enough to prove that two triangles are similar.

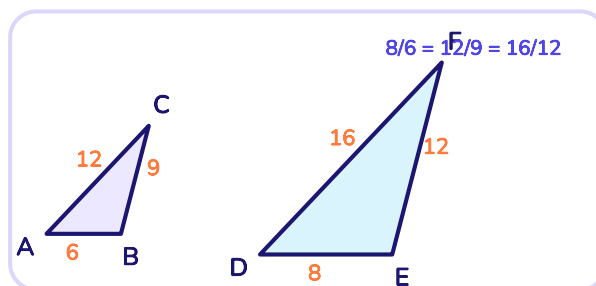
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Similarity criteria for triangles

- **AA:** two pairs of corresponding angles are congruent.
- **SSS:** the three pairs of corresponding sides are proportional.
- **SAS:** two pairs of corresponding sides are proportional and the included angles are congruent.



AA works because the third angles must also match (the angles of a triangle add up to 180°). Compare this with congruence: SSS and SAS ask for *equal* sides, while for similarity they ask for sides in the *same ratio*.



E**Example 3: choosing the right criterion**

Triangle ABC has sides 6, 9, 12 and triangle DEF has sides 8, 12, 16. Compare the sides in increasing order: $\frac{8}{6} = \frac{12}{9} = \frac{16}{12} = \frac{4}{3}$. All three ratios are equal, so $\triangle ABC \sim \triangle DEF$ by SSS, with scale factor $\frac{4}{3}$.

Now suppose a triangle has two sides 5 and 7 with an included angle of 40° , and another has sides 10 and 14 with an included angle of 40° . Since $\frac{10}{5} = \frac{14}{7} = 2$ and the included angles match, the triangles are similar by SAS.

M**Method: deciding if two triangles are similar**

1. Mark everything you know on a sketch: angles, sides, parallel lines.
2. Look for two congruent angle pairs (AA). Shared angles and vertical angles count.
3. If you have sides, order them from shortest to longest and compare the ratios (SSS), or check two sides with their included angle (SAS).
4. Write the similarity statement with the vertices in matching order.

4. The Triangle Proportionality Theorem

When a line cuts across a triangle parallel to one side, it creates a smaller triangle that is similar to the big one, and it slices the other two sides in the same ratio.

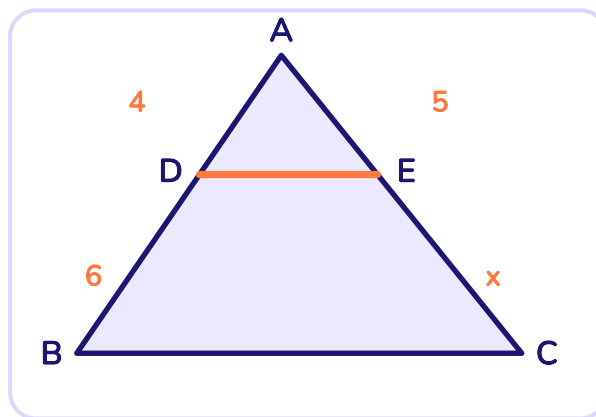
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Triangle Proportionality Theorem

If a line parallel to one side of a triangle meets the other two sides at D and E , then it divides them proportionally:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

The converse is also true: if $\frac{AD}{DB} = \frac{AE}{EC}$, then $DE \parallel BC$.



Why? Because $DE \parallel BC$, the corresponding angles $\angle ADE$ and $\angle ABC$ are congruent, and $\angle A$ is shared, so $\triangle ADE \sim \triangle ABC$ by AA. That gives $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$.

E

Example 4: finding a segment

In the figure, $DE \parallel BC$, $AD = 4$, $DB = 6$, and $AE = 5$. Then $\frac{4}{6} = \frac{5}{x}$, so $4x = 30$ and $x = EC = 7.5$.

If also $BC = 15$, then $\frac{DE}{BC} = \frac{AD}{AB} = \frac{4}{10}$, so $DE = 15 \cdot 0.4 = 6$.



Careful

The ratio $\frac{AD}{DB}$ compares a part to a part. The ratio linking DE and BC needs the *whole* side: $\frac{AD}{AB}$, not $\frac{AD}{DB}$.

5. The Angle Bisector Theorem

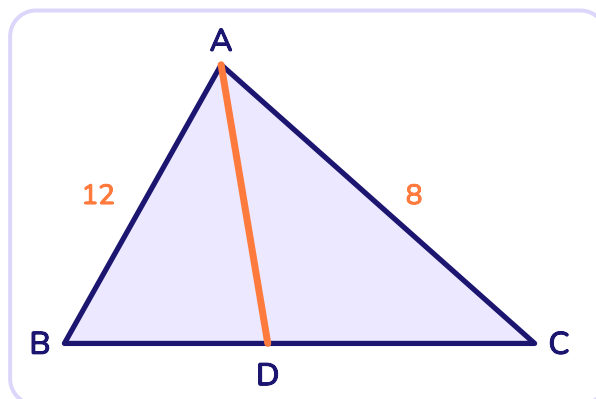
A bisector of an angle of a triangle cuts the opposite side into two pieces. Their lengths are not random.



Angle Bisector Theorem

If AD bisects $\angle A$ of triangle ABC with D on BC , then

$$\frac{BD}{DC} = \frac{AB}{AC}.$$

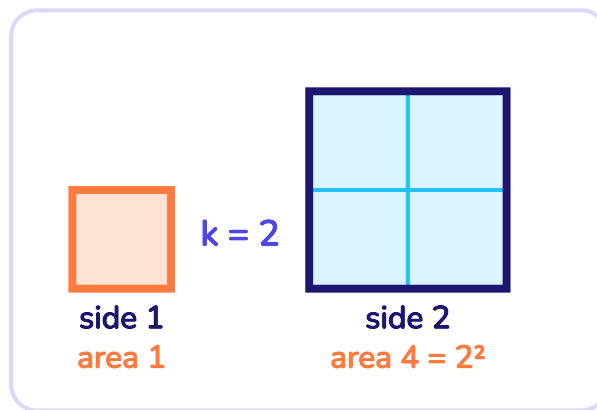


E**Example 5: splitting the third side**

In triangle ABC , $AB = 12$, $AC = 8$, and $BC = 15$. The ratio $AB : AC$ is $12 : 8 = 3 : 2$, so BC is cut into 5 equal shares. Each share is $15 \div 5 = 3$, so $BD = 9$ and $DC = 6$. Check: $\frac{9}{6} = \frac{12}{8} = 1.5$.

6. Scale factor, perimeter, and area

Lengths scale by k , but areas do not. A square with side 1 has area 1. A square with side 2 has area 4, which is 2^2 .

**P****Ratios in similar figures**

If two similar figures have scale factor k , then

- the ratio of any two corresponding lengths (sides, heights, perimeters) is k ;
- the ratio of the areas is k^2 ;
- for similar solids, the ratio of the volumes is k^3 .

E**Example 6: from perimeter to area**

Two similar triangles have perimeters 30 in and 45 in. The scale factor from the small one to the large one is $k = \frac{45}{30} = \frac{3}{2}$. The area ratio is $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$. If the small triangle has area 40 in^2 , the large one has area $40 \cdot \frac{9}{4} = 90 \text{ in}^2$.

!**Zyro's tip**

On my planet we say: “double the sides, quadruple the floor.” If you double every length of a figure, you need four times as much paint to cover it!

7. Writing similarity proofs

A proof is a chain of statements, each backed by a reason. For similarity, you usually prove two angle pairs are congruent and conclude with AA.

M**Method: a two-column similarity proof**

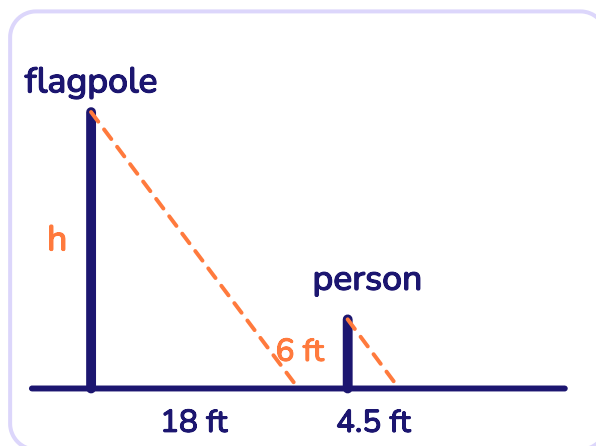
1. State what is *given* and what you must *prove*.
2. List congruent angles with reasons: vertical angles, shared angle, corresponding or alternate interior angles for parallel lines, right angles.
3. Conclude with the criterion (AA, SSS, or SAS) and write the similarity statement in order.
4. Only then use the proportion to get a length.

Given: $DE \parallel BC$ in triangle ABC . **Prove:** $\triangle ADE \sim \triangle ABC$.

Statement	Reason
$DE \parallel BC$	Given
$\angle ADE \cong \angle ABC$	Corresponding angles with parallel lines
$\angle A \cong \angle A$	Shared angle (reflexive property)
$\triangle ADE \sim \triangle ABC$	AA similarity

8. Indirect measurement

Similar triangles let you measure what you cannot reach: the height of a tree, the width of a river, the length of a bridge. At the same moment of the day, the sun's rays are parallel, so an object and its shadow form a right triangle similar to the one formed by any other object and its shadow.



E Example 7: a flagpole

A 6 ft person casts a 4.5 ft shadow while a flagpole casts an 18 ft shadow. The two right triangles are similar by AA (the sun's angle and the right angle), so $\frac{h}{6} = \frac{18}{4.5}$. Then $h = 6 \cdot 4 = 24$ ft, which is about 7.3 m, since $24 \times 0.3048 \approx 7.3$.

Key takeaways

- A dilation with scale factor k multiplies every length by k and keeps angles; at the origin, $(x, y) \mapsto (kx, ky)$.
- Similar figures have congruent corresponding angles and proportional corresponding sides; the order of vertices matters.
- Triangles are similar by AA, SSS, or SAS similarity.
- A line parallel to one side of a triangle gives $\frac{AD}{DB} = \frac{AE}{EC}$; the converse proves lines are parallel.
- An angle bisector gives $\frac{BD}{DC} = \frac{AB}{AC}$.
- With scale factor k : lengths and perimeters scale by k , areas by k^2 , volumes by k^3 .
- In a proof, justify every angle pair, name the criterion, and write the vertices in matching order.



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