

Sequences and Series: math test solutions, College – download the PDF

Test solutions with the detailed point scale. Add up your points and spot what to review.

Suggested time: 45 minutes. Out of 20 points. Calculator allowed only when the problem says so.

1 Limits of sequences

/ 4 pts

(a) Divide by n^2 : the limit is $\frac{4}{2} = 2$. (1 pt)

(b) $\frac{n}{n+1} \rightarrow 1$, so the even terms tend to 1 and the odd terms to -1 . The sequence diverges. (1 pt)

(c) Multiply by the conjugate: $c_n = \frac{n}{\sqrt{n^2 + n} + n} = \frac{1}{\sqrt{1 + \frac{1}{n}} + 1}$ (1 pt), which tends to $\frac{1}{2}$. (1 pt)

2 Geometric series

/ 3 pts

(a) $r = -\frac{2}{3}$, $|r| < 1$, $\text{sum} = \frac{5}{1 + \frac{2}{3}} = 3$. (1 pt)

(b) $0.45 + 0.0045 + \dots: \frac{0.45}{0.99} = \frac{45}{99} = \frac{5}{11}$. (1 pt)

(c) It converges when $|\frac{x}{4}| < 1$, i.e. $-4 < x < 4$, and the sum is $\frac{1}{1 - \frac{x}{4}} = \frac{4}{4 - x}$. (1 pt)

3 Choosing a test

/ 4 pts

(a) Limit comparison with $\frac{1}{n^2}$: $\frac{5n^3}{n^3 + 2} \rightarrow 5$, and $\sum \frac{1}{n^2}$ converges, so the series converges. (2 pts)

(b) $\frac{n}{5n-1} \rightarrow \frac{1}{5} \neq 0$: the series diverges by the divergence test. (1 pt)

(c) $\frac{1}{n\sqrt{n}} = n^{-3/2}$: a p-series with $p = 1.5 > 1$, so it converges. (1 pt)

4 Ratio and root tests

/ 4 pts

(a) $\frac{a_{n+1}}{a_n} = \frac{2}{n+1} \rightarrow 0 < 1$ (1 pt), so the series converges. (1 pt)

(b) $\sqrt[n]{a_n} = \frac{n}{2n+1} \rightarrow \frac{1}{2} < 1$ (1 pt), so the series converges. (1 pt)

5 An alternating series

/ 3 pts

The terms $\frac{1}{2n+1}$ are positive, decreasing and tend to 0, so the series converges. (1 pt)

Limit comparison of $\sum \frac{1}{2n+1}$ with $\sum \frac{1}{n}$ gives the limit $\frac{1}{2}$, so the absolute series diverges. (1 pt)

The series converges conditionally. (1 pt)

6

Power and Taylor series

/ 2 pts

(a) Put $u = -x^2$: $\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} = 1 - x^2 + x^4 - \dots$. (1 pt)

(b) It converges when $|-x^2| < 1$, so $|x| < 1$: $R = 1$. (1 pt)

**Test yourself: quick challenge for College**

Change level

Level : College

6 random questions, instant feedback.

**Start the challenge****Keep exploring with Zyro****Math tests**

Sequences and Series: math test,
College

**Math lessons**

Sequences and Series: math lesson,
College

 Math quizzes

Sequences and Series: math quiz,
College

 Math tests

Linear Systems and Matrices: math test,
College

 Math tests

Vector Spaces and Linear
Transformations: math test, College

 Math lessons

Linear Systems and Matrices: math
lesson, College

Zyro takes you to our partner sites

Blast off to other planets of knowledge and their apps.

Lift off