

# Sequences, Series, and Induction: practice solutions, Grade 12 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

## 1 Terms of an arithmetic sequence



Add 6 each time: 4, 10, 16, 22, 28.

$$a_{12} = a_1 + 11d = 4 + 11 \times 6 = 70.$$

**Answer:** the first five terms are 4, 10, 16, 22, 28 and  $a_{12} = 70$ .

## 2 Arithmetic, geometric, or neither?



(a) Ratios:  $9/3 = 27/9 = 81/27 = 3$ , so it is **geometric** with  $r = 3$ .

(b) Differences:  $-7$  each time, so it is **arithmetic** with  $d = -7$ .

(c) Differences are 3, 5, 7 (not constant) and ratios  $5/2, 10/5 = 2$  (not constant), so it is **neither**.

## 3 Expand a sum



Replace  $k$  by 1, 2, 3, 4, 5:  $1 + 4 + 7 + 10 + 13 = 35$ .

**Answer:** 35.

#### 4 Fifth term of a geometric sequence



$$a_5 = a_1 r^4 = 2 \times 5^4 = 2 \times 625 = 1250.$$

Answer:  $a_5 = 1250$ .

#### 5 Does the series have a sum?



(a) **False.** The ratio is  $r = 2$ , and  $|r| \geq 1$ , so the series diverges.

(b) **True.** The ratio is  $r = 0.2$  with  $|r| < 1$ . The sum is  $\frac{5}{1 - 0.2} = \frac{5}{0.8} = 6.25$ .

#### 6 Recursive to explicit



Terms: 5, 13, 21, 29. Each term is 8 more than the last, so the sequence is arithmetic with  $d = 8$ .

$$a_n = 5 + (n - 1) \cdot 8 = 8n - 3. \text{ Check: } a_4 = 32 - 3 = 29.$$

#### 7 Expand with Pascal's triangle



Row 5 is 1, 5, 10, 10, 5, 1. Since 1 raised to any power is 1:

$$(x + 1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1.$$

Check with  $x = 1$ :  $2^5 = 32 = 1 + 5 + 10 + 10 + 5 + 1$ .

#### 8 Stadium seating



This is arithmetic with  $a_1 = 22$ ,  $d = 3$ ,  $n = 18$ .

$$a_{18} = 22 + 17 \times 3 = 73.$$

$$S_{18} = \frac{18(22 + 73)}{2} = 9 \times 95 = 855.$$

Answer: the last row has 73 seats, and the section has 855 seats.

### 9 Weekly savings



Arithmetic with  $a_1 = 50$ ,  $d = 5$ ,  $n = 26$ .

$$a_{26} = 50 + 25 \times 5 = 175.$$

$$S_{26} = \frac{26(50 + 175)}{2} = 13 \times 225 = 2925.$$

**Answer:** 175 dollars in week 26 and 2,925 dollars in total.

### 10 A spreading rumor



The daily counts 6, 12, 24, ... form a geometric sequence with  $a_1 = 6$ ,  $r = 2$ .

$$S_{10} = 6 \cdot \frac{2^{10} - 1}{2 - 1} = 6 \times 1023 = 6138.$$

**Answer:** 6,138 students (a large school indeed!).

### 11 How many terms?



(a)  $r = 4$ . Solve  $3 \cdot 4^{n-1} = 3072$ :  $4^{n-1} = 1024 = 4^5$ , so  $n - 1 = 5$  and  $n = 6$ .

$$(b) S_6 = 3 \cdot \frac{4^6 - 1}{4 - 1} = 4096 - 1 = 4095.$$

Check:  $3 + 12 + 48 + 192 + 768 + 3072 = 4095$ .

### 12 Repeating decimal as a fraction



$0.4545\ldots = 0.45 + 0.0045 + 0.000045 + \cdots$ , so  $a_1 = 0.45$  and  $r = 0.01$ .

$$S = \frac{0.45}{1 - 0.01} = \frac{0.45}{0.99} = \frac{45}{99} = \frac{5}{11}.$$

Check:  $5 \div 11 = 0.4545\ldots$

### 13 Using the sum rules



(a)  $3 \cdot \frac{40 \cdot 41}{2} + 2 \cdot 40 = 3 \times 820 + 80 = 2540.$

(b)  $\frac{12 \cdot 13 \cdot 25}{6} = 650.$

### 14 A Fibonacci-style sequence



Terms: 1, 3, 4, 7, 11, 18, 29, 47, 76.

So  $f_9 = 76$ . Since  $f_8 = 47 \leq 50$  and  $f_9 = 76 > 50$ , the first index is  $n = 9$ .

### 15 Two coefficients in a binomial expansion



(a) The general term is  $\binom{6}{k} (2x)^k (-1)^{6-k}$ . For  $x^3$ :  $k = 3$ , giving  $20 \cdot 8 \cdot (-1)^3 = -160$ .

(b) The general term is  $\binom{6}{k} (x^2)^{6-k} (x^{-1})^k = \binom{6}{k} x^{12-3k}$ . The exponent is 0 when  $k = 4$ , so the constant term is  $\binom{6}{4} = 15$ .

### 16 Induction: a sum of products



Base case.  $n = 1$ : left side  $1 \cdot 2 = 2$ , right side  $\frac{1 \cdot 2 \cdot 3}{3} = 2$ .

Inductive step. Assume the formula for  $n = k$ . Then the sum up to  $k + 1$  is

$$\frac{k(k+1)(k+2)}{3} + (k+1)(k+2) = (k+1)(k+2) \left( \frac{k}{3} + 1 \right) = \frac{(k+1)(k+2)(k+3)}{3}$$

which is the formula for  $n = k + 1$ .

By induction the formula holds for all  $n \geq 1$ .

### 17 Induction: divisibility



**Base case.**  $4^1 - 1 = 3$ , divisible by 3.

**Inductive step.** Assume  $4^k - 1 = 3m$  for some integer  $m$ . Then

$4^{k+1} - 1 = 4 \cdot 4^k - 1 = 4(4^k - 1) + 3 = 4(3m) + 3 = 3(4m + 1)$ , which is divisible by 3.

By induction, 3 divides  $4^n - 1$  for all  $n \geq 1$ .

### 18 Find the formula, then prove it



(a)  $a_2 = 7, a_3 = 17, a_4 = 37$ . The formula gives  $5 \cdot 2 - 3 = 7, 5 \cdot 4 - 3 = 17, 5 \cdot 8 - 3 = 37$ , as expected.

(b) **Base case.**  $5 \cdot 2^0 - 3 = 2 = a_1$ .

**Inductive step.** If  $a_k = 5 \cdot 2^{k-1} - 3$ , then

$a_{k+1} = 2a_k + 3 = 10 \cdot 2^{k-1} - 6 + 3 = 5 \cdot 2^k - 3$ , the formula for  $k + 1$ .

Hence  $a_n = 5 \cdot 2^{n-1} - 3$  for all  $n \geq 1$ .

### 19 The bouncing ball



The ball falls 10 ft once. Then, for each rebound, it goes up and comes down: the rebound heights are **6, 3.6, 2.16, ...**, a geometric sequence with  $a_1 = 6, r = 0.6$ .

Sum of the rebound heights:  $\frac{6}{1 - 0.6} = 15$  ft. Each is traveled twice:  $2 \times 15 = 30$  ft.

Total:  $10 + 30 = 40$  ft, which is  $40 \times 0.3048 = 12.192 \approx 12.2$  m.

### 20 Finding $a_1$ and $d$



From  $a_9 - a_4 = 5d = 25$ , we get  $d = 5$ . Then  $a_1 = a_4 - 3d = 17 - 15 = 2$ .

$a_{30} = 2 + 29 \times 5 = 147$ , so  $S_{30} = \frac{30(2 + 147)}{2} = 15 \times 149 = 2235$ .

## 21 An unknown ratio



$$a_5 = 81r^4 = 16, \text{ so } r^4 = \frac{16}{81} \text{ and } r = \frac{2}{3} \text{ or } r = -\frac{2}{3}.$$

For  $r = \frac{2}{3}$ : terms 81, 54, 36, 24, 16, and  $S_5 = 211$ .

For  $r = -\frac{2}{3}$ : terms 81, -54, 36, -24, 16, and  $S_5 = 55$ .

Check with the formula for  $r = \frac{2}{3}$ :  $81 \cdot \frac{1 - 32/243}{1/3} = 243 \cdot \frac{211}{243} = 211$ .



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