

Sequences and Series: practice solutions, Grade 11 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Name that sequence



- a. Differences: **7, 7, 7**. Arithmetic with $d = 7$.
- b. Ratios: **2, 2, 2**. Geometric with $r = 2$.
- c. Differences **2, 3, 4** and ratios **2, 1.75, ...** are not constant. Neither.
- d. Ratios: $\frac{27}{81} = \frac{1}{3}$, $\frac{9}{27} = \frac{1}{3}$, $\frac{3}{9} = \frac{1}{3}$. Geometric with $r = \frac{1}{3}$.

2 First terms of an arithmetic sequence



- (a) **9, 13, 17, 21, 25**.
- (b) $a_n = 9 + (n - 1) \cdot 4 = 4n + 5$.
- (c) $a_{12} = 4 \cdot 12 + 5 = 53$.

3 First terms of a geometric sequence



5, 15, 45, 135, 405.

$$a_7 = 5 \cdot 3^6 = 5 \cdot 729 = 3,645.$$

4 Explicit to recursive



- (a) $5, 3, 1, -1$.
- (b) The difference is -2 : $a_1 = 5$ and $a_{n+1} = a_n - 2$.
- (c) $a_{15} = 7 - 30 = -23$.

5 Your first sigma sum



The terms are $5, 7, 9, 11$. The sum is $5 + 7 + 9 + 11 = 32$.

Check: $\frac{4(5 + 11)}{2} = 32$.

6 Even numbers up to 50



The sequence is arithmetic with $a_1 = 2, d = 2, a_n = 50$. Then $2n = 50$ gives $n = 25$ terms.

$$S_{25} = \frac{25(2 + 50)}{2} = 25 \cdot 26 = 650.$$

7 Converge or diverge?



(a) $r = \frac{1}{3}, |r| < 1$, so $S = \frac{6}{1 - \frac{1}{3}} = \frac{6}{\frac{2}{3}} = 9$.

(b) $r = 2, |r| \geq 1$: the series diverges and has no sum.

8 Which term is it?



The formula is $a_n = 4 + (n - 1) \cdot 5 = 5n - 1$.

(a) $5n - 1 = 139$ gives $5n = 140$, so $n = 28$. It is the 28th term.

(b) $5n - 1 = 200$ gives $n = 40.2$, which is not a whole number. So 200 is not a term.

9 Finding the ratio from two terms



From a_2 to a_5 there are 3 steps, so $r^3 = \frac{324}{12} = 27$ and $r = 3$.

$a_1 = \frac{12}{3} = 4$. The formula is $a_n = 4 \cdot 3^{n-1}$.

Check: $a_5 = 4 \cdot 81 = 324$.

10 Two terms of an arithmetic sequence



Five steps separate a_4 and a_9 : $5d = 42 - 17 = 25$, so $d = 5$.

$a_1 = 17 - 3 \cdot 5 = 2$, and $a_n = 2 + (n - 1) \cdot 5 = 5n - 3$.

$a_{20} = 97$, so $S_{20} = \frac{20(2 + 97)}{2} = 10 \cdot 99 = 990$.

11 A geometric sum



$a_1 = 4$ and $r = 3$.

$S_7 = 4 \cdot \frac{1 - 3^7}{1 - 3} = 4 \cdot \frac{-2186}{-2} = 4 \cdot 1093 = 4,372$.

12 Theater seats



The row counts form an arithmetic sequence with $a_1 = 14$, $d = 2$.

(a) $a_{18} = 14 + 17 \cdot 2 = 48$ seats.

(b) $S_{18} = \frac{18(14 + 48)}{2} = 9 \cdot 62 = 558$.

The theater has 558 seats.

13 A sigma sum with powers



The terms are 6, 12, 24, 48, 96, a geometric series with $a_1 = 6$, $r = 2$, 5 terms.

$$S_5 = 6 \cdot \frac{1 - 2^5}{1 - 2} = 6 \cdot 31 = 186.$$

Check by adding: $6 + 12 + 24 + 48 + 96 = 186$.

14 A repeating decimal as a fraction



$0.363636 \dots = 0.36 + 0.0036 + 0.000036 + \dots$, with $a_1 = 0.36$ and $r = 0.01$.

$$S = \frac{0.36}{1 - 0.01} = \frac{0.36}{0.99} = \frac{36}{99} = \frac{4}{11}.$$

15 Saving for a bike



Deposits form an arithmetic sequence with $a_1 = 50$, $d = 15$. So

$$S_n = \frac{n[100 + 15(n - 1)]}{2} = \frac{15n^2 + 85n}{2}.$$

We need $15n^2 + 85n \geq 4000$, that is $3n^2 + 17n - 800 \geq 0$. The positive root is

$$n = \frac{-17 + \sqrt{9889}}{6} \approx 13.7.$$

$$\text{Test: } S_{13} = \frac{13 \cdot 280}{2} = 1,820 < 2,000 \text{ and } S_{14} = \frac{14 \cdot 295}{2} = 2,065 \geq 2,000.$$

She reaches her goal in month 14.

16 The bouncing ball



Down: 20 ft. Then each rebound height is traveled twice (up and down). The rebound heights are 12, 7.2, 4.32, ..., geometric with $a_1 = 12$, $r = 0.6$.

$$\text{Sum of rebounds: } \frac{12}{1 - 0.6} = 30 \text{ ft.}$$

Total distance: $20 + 2 \cdot 30 = 80$ ft, about 24.4 m.

17 Yearly deposits



The first deposit grows for 10 years and the last one for 1 year. The total is

$500(1.04) + 500(1.04)^2 + \cdots + 500(1.04)^{10}$, a geometric series with

$$a_1 = 500 \cdot 1.04 = 520, r = 1.04, n = 10.$$

$$S_{10} = 520 \cdot \frac{1.04^{10} - 1}{0.04} \approx 520 \cdot 12.0061 \approx 6,243.18 \text{ dollars.}$$

18 Three numbers in an arithmetic sequence



Write them $m - d$, m , $m + d$. The sum is $3m = 27$, so $m = 9$.

The product is $9(81 - d^2) = 693$, so $81 - d^2 = 77$ and $d^2 = 4$, $d = \pm 2$.

The numbers are **7, 9, 11** (or **11, 9, 7**). Check: $7 + 9 + 11 = 27$ and $7 \cdot 9 \cdot 11 = 693$.

19 Solving for x



(a) It is geometric with $a_1 = 8$, $r = x$. It converges exactly when $|x| < 1$, that is $-1 < x < 1$.

(b) $\frac{8}{1-x} = 20$, so $1-x = 0.4$ and $x = 0.6$. This satisfies $|x| < 1$.

20 Geometric means



We need $a_1 = 5$ and $a_5 = 80$, so $5r^4 = 80$, $r^4 = 16$, $r = 2$ or $r = -2$.

If $r = 2$: **5, 10, 20, 40, 80**. If $r = -2$: **5, -10, 20, -40, 80**.

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An alternating series

(a) Terms: 9, -4.5, 2.25, -1.125. $S_4 = 5.625$. Formula:

$$9 \cdot \frac{1 - (1/16)}{3/2} = 9 \cdot \frac{15/16}{3/2} = 5.625.$$

(b) $r = -\frac{1}{2}$, $|r| < 1$: $S = \frac{9}{1 + \frac{1}{2}} = 6$.

(c) $6 - 5.625 = 0.375$.


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