

Sequences and Series: practice solutions, College – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Limit of a rational sequence



Divide the numerator and denominator by n : $\frac{5 - \frac{3}{n}}{2 + \frac{7}{n}}$. Since $\frac{3}{n} \rightarrow 0$ and $\frac{7}{n} \rightarrow 0$, the limit is $\frac{5}{2}$.

2 Sum of a geometric series



The first term is $a = 3$ and each term is half the previous one, so $r = \frac{1}{2}$. Since $|r| < 1$, the sum is $\frac{3}{1 - \frac{1}{2}} = 6$.

3 Use the divergence test



$\frac{n}{n+4} = \frac{1}{1 + \frac{4}{n}} \rightarrow 1 \neq 0$. By the divergence test, the series diverges.

4 Classify three p-series



These are p-series. (a) $p = 3 > 1$: converges. (b) $\frac{1}{\sqrt{n}} = n^{-1/2}$, so $p = \frac{1}{2} \leq 1$: diverges. (c) $p = \frac{2}{3} \leq 1$: diverges.

5 A repeating decimal



$0.272727\ldots = 0.27 + 0.0027 + 0.000027 + \cdots$, with $a = 0.27$ and $r = 0.01$. The sum is $\frac{0.27}{1 - 0.01} = \frac{0.27}{0.99} = \frac{27}{99} = \frac{3}{11}$.

6 A telescoping partial sum



The terms telescope: $s_N = (1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + \cdots + (\frac{1}{N} - \frac{1}{N+1}) = 1 - \frac{1}{N+1}$. So $s_4 = 1 - \frac{1}{5} = \frac{4}{5}$. As $N \rightarrow \infty$, $s_N \rightarrow 1$, so the sum is 1.

7 Radius of a simple power series



This is geometric with ratio $r = \frac{x}{2}$. It converges when $|\frac{x}{2}| < 1$, that is $|x| < 2$, so $R = 2$. At $x = 1$ the sum is $\frac{1}{1 - \frac{1}{2}} = 2$.

8 A bouncing ball



The first fall is 10 ft. The rebound heights are **6, 3.6, 2.16, ...**, a geometric sequence with $a = 6$ and $r = 0.6$; each rebound is traveled twice (up and down).

Sum of rebound heights: $\frac{6}{1 - 0.6} = 15$ ft. Total: $10 + 2(15) = 40$ ft, which is about 12.2 m.

9 Ratio test with powers



$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{(n+1)^2}{3^{n+1}} \cdot \frac{3^n}{n^2} = \frac{1}{3} \left(\frac{n+1}{n} \right)^2 \rightarrow \frac{1}{3} < 1. \text{ The series converges absolutely.}$$

10 Two comparisons



(a) For $n \geq 1$, $\frac{1}{n^2 + 5} < \frac{1}{n^2}$, and $\sum \frac{1}{n^2}$ converges, so the series converges.

(b) Compare with $b_n = \frac{1}{n}$: $\frac{a_n}{b_n} = \frac{n}{2n-1} \rightarrow \frac{1}{2}$, a finite positive number. Since $\sum \frac{1}{n}$ diverges, so does $\sum \frac{1}{2n-1}$.

11 Integral test with an exponential



Let $f(x) = xe^{-x^2}$. It is positive and continuous. Its derivative is

$f'(x) = (1 - 2x^2)e^{-x^2} < 0$ for $x \geq 1$, so f is decreasing.

$$\int_1^\infty xe^{-x^2} dx = \left[-\frac{1}{2}e^{-x^2} \right]_1^\infty = \frac{1}{2e} \approx 0.184, \text{ which is finite. So the series converges.}$$

12 How many terms are enough?



The terms $\frac{1}{n^2}$ decrease to 0, so the error after N terms is at most $b_{N+1} = \frac{1}{(N+1)^2}$.

For $N = 2$: $\frac{1}{9} \approx 0.111$, too big. For $N = 3$: $\frac{1}{16} = 0.0625 < 0.1$. So three terms suffice:

$$s_3 = 1 - \frac{1}{4} + \frac{1}{9} = \frac{31}{36} \approx 0.861.$$

13 Absolute or conditional?



The terms $\frac{1}{\sqrt{n}}$ decrease to 0, so the alternating series test shows convergence. The absolute series is $\sum n^{-1/2}$, a p-series with $p = \frac{1}{2} \leq 1$, which diverges. So the series is conditionally convergent.

14 A Maclaurin approximation



Replace x by $2x$ in e^x :

$$e^{2x} = 1 + 2x + \frac{(2x)^2}{2} + \frac{(2x)^3}{6} + \dots = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \dots$$

At $x = 0.1$: $1 + 0.2 + 0.02 + 0.00133 = 1.22133$. The exact value is $e^{0.2} \approx 1.22140$, so the error is less than 0.0001.

15 True or false?



(a) False: $a_n = \frac{1}{n} \rightarrow 0$ but the harmonic series diverges. (b) True: this is the divergence test. (c) True: absolute convergence implies convergence. (d) False: the terms ± 1 do not tend to 0, and the partial sums keep switching between two values (-1 and 0 if the sum starts at $n = 1$).

16 Interval of convergence



Ratio test: $\left| \frac{a_{n+1}}{a_n} \right| = \frac{|x|}{3} \cdot \frac{n}{n+1} \rightarrow \frac{|x|}{3}$. The series converges for $|x| < 3$, so $R = 3$.

At $x = 3$: $\sum \frac{1}{n}$, the harmonic series, diverges. At $x = -3$: $\sum \frac{(-1)^n}{n}$, which converges by the alternating series test. The interval is $[-3, 3)$.

17 A telescoping series with partial fractions



Partial fractions: $\frac{2}{n(n+2)} = \frac{1}{n} - \frac{1}{n+2}$. The partial sum telescopes, leaving

$$s_N = 1 + \frac{1}{2} - \frac{1}{N+1} - \frac{1}{N+2}.$$

As $N \rightarrow \infty$, $s_N \rightarrow \frac{3}{2}$. The sum is $\frac{3}{2}$.

18 When the ratio test fails

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For $\frac{1}{n}$: $\frac{n}{n+1} \rightarrow 1$. For $\frac{1}{n^2}$: $\frac{n^2}{(n+1)^2} \rightarrow 1$. Both give $L = 1$.

But $\sum \frac{1}{n}$ diverges and $\sum \frac{1}{n^2}$ converges (p-series). So when $L = 1$ the series can do either: the ratio test is inconclusive.

19 Root test

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The root test gives $\sqrt[n]{|a_n|} = \frac{2n+1}{3n+4} \rightarrow \frac{2}{3} < 1$. The series converges absolutely.

20 Estimating a logarithm

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With $x = 0.1$: $0.1 - 0.005 + 0.000333 = 0.095333$.

The series alternates with decreasing terms, so the error is at most the next term,

$\frac{0.1^4}{4} = 0.000025$. The true value **0.095310** indeed differs by about **0.000023**.

21 Differentiating a power series

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Differentiate term by term: $\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} nx^{n-1}$. Multiply by x : $\sum nx^n = \frac{x}{(1-x)^2}$.

At $x = \frac{1}{2}$: $\frac{1/2}{(1/2)^2} = 2$. So $\sum \frac{n}{2^n} = 2$.

22 Factorial and power

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$\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = \frac{n^n}{(n+1)^n} = \left(1 + \frac{1}{n}\right)^{-n} \rightarrow \frac{1}{e} \approx 0.368 < 1$. The series converges.



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