

Right Triangles and Trigonometry: math lesson, Grade 10 – download the PDF

A ladder against a wall, a wheelchair ramp, a drone hovering over a field, a surveyor measuring a river without getting wet: all of these hide **triangles**. In this chapter you will learn to find missing lengths with the Pythagorean theorem, to use shortcuts in special triangles, to turn angles into lengths with **sine, cosine and tangent**, and to go beyond right triangles with the Law of Sines and the Law of Cosines.

1. The Pythagorean Theorem and Its Converse

D Right triangle

A right triangle has one 90° angle. The two sides that form it are the **legs**. The side opposite the right angle is the **hypotenuse**, and it is always the longest side.

P Pythagorean theorem

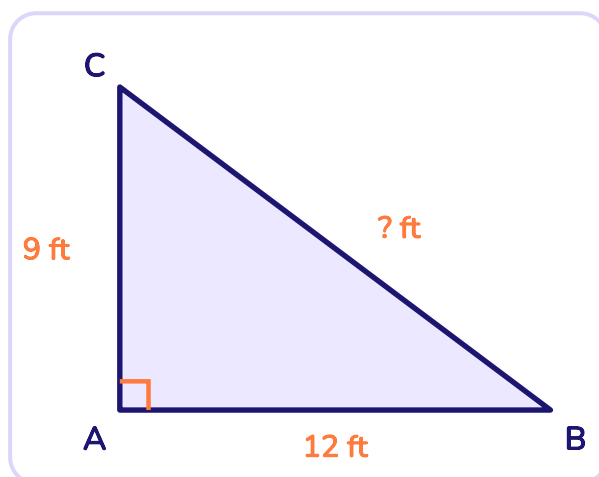
If a triangle has legs a and b and hypotenuse c , then

$$a^2 + b^2 = c^2.$$

E Example 1: finding a hypotenuse

A ramp has a horizontal run of 12 ft and a vertical rise of 9 ft. How long is the ramp surface?

$$c^2 = 12^2 + 9^2 = 144 + 81 = 225, \text{ so } c = \sqrt{225} = 15 \text{ ft (about 4.57 m).}$$



To find a leg instead, subtract: $a^2 = c^2 - b^2$. Some whole-number triples show up so often that they are worth memorizing:

Triple (a, b, c)	Check
$(3, 4, 5)$	$9 + 16 = 25$
$(5, 12, 13)$	$25 + 144 = 169$
$(8, 15, 17)$	$64 + 225 = 289$
$(7, 24, 25)$	$49 + 576 = 625$
$(20, 21, 29)$	$400 + 441 = 841$

P**Converse and classification**

Let c be the longest side of a triangle with sides a, b, c .

- If $a^2 + b^2 = c^2$, the triangle is **right**.
- If $a^2 + b^2 > c^2$, the triangle is **acute**.
- If $a^2 + b^2 < c^2$, the triangle is **obtuse**.

E**Example 2: testing a triangle**

Is a triangle with sides 7, 9 and 12 right, acute or obtuse?

The longest side is 12. $7^2 + 9^2 = 49 + 81 = 130$ and $12^2 = 144$. Since $130 < 144$, the triangle is **obtuse**.

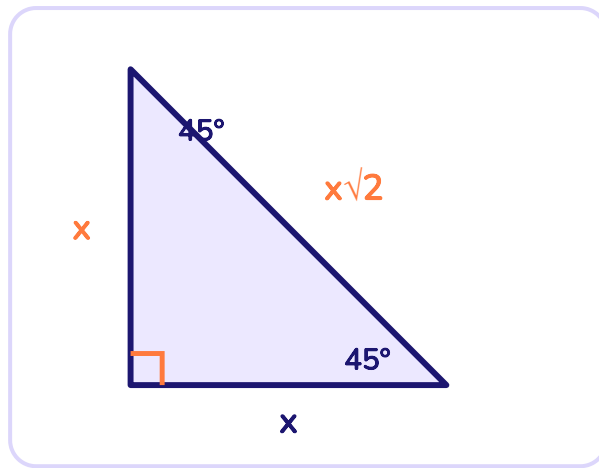
2. Special Right Triangles

Two right triangles are so common that you can skip the theorem and use their fixed side ratios.

P**45-45-90 triangle**

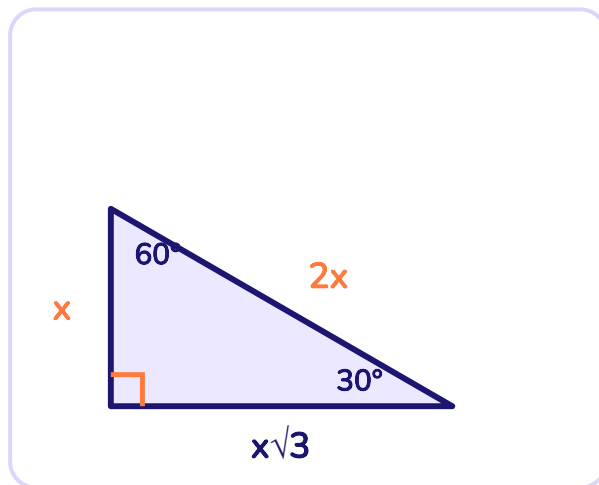
Both legs are equal, say x . By the Pythagorean theorem the hypotenuse is

$$\sqrt{x^2 + x^2} = x\sqrt{2}.$$



P 30-60-90 triangle

Cut an equilateral triangle of side $2x$ in half. The short leg (opposite 30°) is x , the hypotenuse is $2x$, and the long leg (opposite 60°) is $x\sqrt{3}$.



E**Example 3: using the ratios**

(a) A square tile has a diagonal of 14 in. The diagonal splits it into two 45-45-90 triangles, so a side is $\frac{14}{\sqrt{2}} = 7\sqrt{2} \approx 9.90$ in.

(b) A 30-60-90 triangle has a hypotenuse of 18 cm. The short leg is $18 \div 2 = 9$ cm and the long leg is $9\sqrt{3} \approx 15.59$ cm.

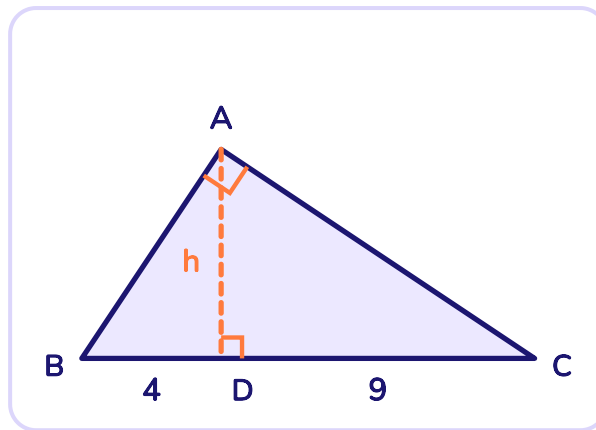
**Watch out**

The $\sqrt{3}$ goes with the **long** leg, never with the hypotenuse. The hypotenuse is always the biggest side, so $2x$ beats $x\sqrt{3} \approx 1.73x$.

3. The Geometric Mean in Right Triangles**D****Geometric mean**

The geometric mean of two positive numbers p and q is the number x such that $\frac{p}{x} = \frac{x}{q}$. Equivalently, $x = \sqrt{pq}$.

Draw the altitude from the right angle to the hypotenuse. It cuts the big right triangle into two smaller ones, and all three triangles are **similar**. Similar triangles give proportions, and these proportions are geometric means.



P

Geometric mean theorems

- The altitude is the geometric mean of the two segments of the hypotenuse:
 $h^2 = p \cdot q$.
- Each leg is the geometric mean of the hypotenuse and the segment next to that leg: $a^2 = p \cdot c$ and $b^2 = q \cdot c$.

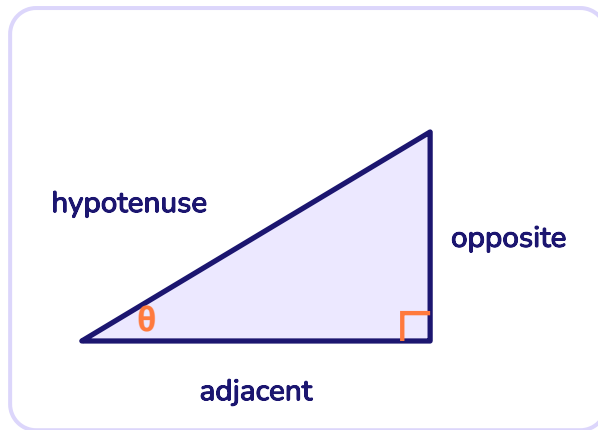
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Example 4: the altitude

In the figure the segments are $p = 4$ and $q = 9$. Then $h = \sqrt{4 \cdot 9} = 6$. The hypotenuse is $c = 13$, so $AB = \sqrt{4 \cdot 13} = 2\sqrt{13} \approx 7.21$ and $AC = \sqrt{9 \cdot 13} = 3\sqrt{13} \approx 10.82$. Check: $52 + 117 = 169 = 13^2$.

4. Sine, Cosine, and Tangent

Two right triangles with the same acute angle are similar, so the ratio of two sides depends only on the angle. These ratios get names.



D Trigonometric ratios

For an acute angle θ in a right triangle:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}.$$

Memory trick: **SOH-CAH-TOA**.

M Method: finding a side

1. Mark the angle, then label the sides opposite, adjacent and hypotenuse.
2. Pick the ratio that uses the side you know and the side you want.
3. Write the equation and solve for the unknown.
4. Check that your calculator is in **degree** mode.

E

Example 5: ratios

A right triangle has legs 8 (adjacent to A) and 15 (opposite A), and hypotenuse 17.

Then $\sin A = \frac{15}{17}$, $\cos A = \frac{8}{17}$, $\tan A = \frac{15}{8}$.

E

Example 6: finding a side

A cable of length 12 m makes a 35° angle with the ground. Its height is

$12 \sin 35^\circ \approx 12(0.5736) \approx 6.88$ m. If instead the adjacent leg is 10 ft and the angle is 52° , the opposite leg is $10 \tan 52^\circ \approx 12.80$ ft.

Special values worth knowing:

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Also remember that the two acute angles of a right triangle are complementary, so $\sin A = \cos(90^\circ - A)$.

5. Inverse Trigonometric Functions

When you know two sides and want the **angle**, use the inverse functions $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ (also written arcsin, arccos, arctan).

P**Inverse ratios**

If $\sin \theta = \frac{\text{opp}}{\text{hyp}}$, then $\theta = \sin^{-1} \left(\frac{\text{opp}}{\text{hyp}} \right)$. The same idea works for cosine and tangent.

E**Example 7: finding an angle**

A right triangle has legs 5 and 12. Then $\tan \theta = \frac{5}{12}$, so $\theta = \tan^{-1} \left(\frac{5}{12} \right) \approx 22.6^\circ$.

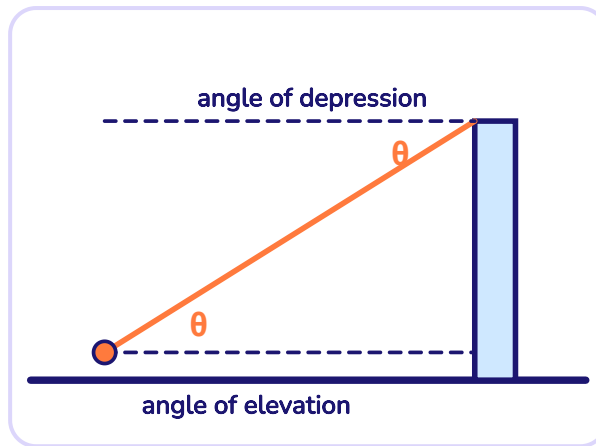
The other acute angle is $90^\circ - 22.6^\circ = 67.4^\circ$.

**Watch out**

$\sin^{-1} x$ does **not** mean $\frac{1}{\sin x}$. It is the angle whose sine is x .

6. Angles of Elevation and Depression**D****Elevation and depression**

The **angle of elevation** is measured upward from the horizontal to a line of sight. The **angle of depression** is measured downward from the horizontal.



The two horizontal lines are parallel, so the angle of elevation from the observer equals the angle of depression from the top (alternate interior angles).

E Example 8: a tower

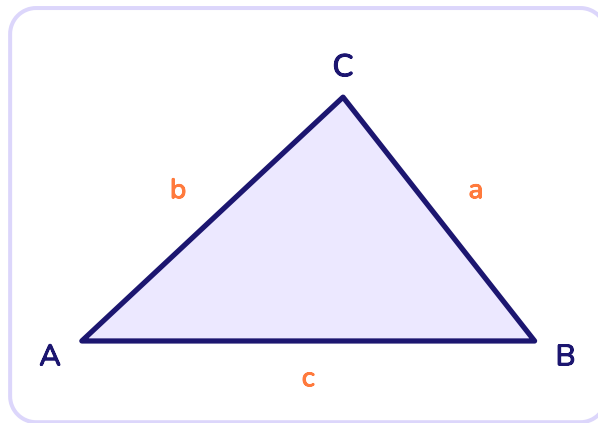
You stand 40 m from the base of a tower. The angle of elevation to the top is 62° and your eyes are 1.6 m above the ground. The tower height is $40 \tan 62^\circ + 1.6 \approx 75.23 + 1.6 \approx 76.8$ m.

E Example 9: a cliff

From the top of a 90 ft cliff, the angle of depression to a boat is 18° . The boat is $\frac{90}{\tan 18^\circ} \approx 277.0$ ft from the base of the cliff.

7. The Law of Sines and the Law of Cosines

Not every triangle has a right angle. For any triangle with sides a, b, c opposite angles A, B, C :



P Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

P Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

When $C = 90^\circ$, $\cos C = 0$ and you recover the Pythagorean theorem.

M Which law should you use?

- Two angles and a side (AAS or ASA): Law of Sines.
- Two sides and the included angle (SAS), or three sides (SSS): Law of Cosines.



Example 10: Law of Sines

In a triangle, $A = 40^\circ$, $B = 65^\circ$ and $a = 10$. Then $b = \frac{10 \sin 65^\circ}{\sin 40^\circ} \approx 14.10$.



Example 11: Law of Cosines

Given $a = 8$, $b = 11$ and $C = 50^\circ$: $c^2 = 64 + 121 - 176 \cos 50^\circ \approx 71.87$, so $c \approx 8.48$. For the sides 7, 9 and 12 from Example 2, $\cos C = \frac{49 + 81 - 144}{2 \cdot 7 \cdot 9} = -\frac{1}{9}$, so $C \approx 96.4^\circ$, an obtuse angle, as predicted.



Zyro's tip

A negative cosine means an obtuse angle. If your calculator gives a cosine below zero, do not panic: the angle is simply larger than 90° .



Watch out

With two sides and a *non-included* angle (SSA), the Law of Sines can give two possible triangles. Always check whether the second solution fits.

Key takeaways

- Right triangle: $a^2 + b^2 = c^2$. Converse: if the equality holds, the triangle is right; compare $a^2 + b^2$ with c^2 to classify it as acute or obtuse.
- 45-45-90: sides $x, x, x\sqrt{2}$. 30-60-90: sides $x, x\sqrt{3}, 2x$.
- Altitude to the hypotenuse: $h^2 = pq, a^2 = pc, b^2 = qc$.
- SOH-CAH-TOA: sine = opposite/hypotenuse, cosine = adjacent/hypotenuse, tangent = opposite/adjacent.
- Use $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ to find an angle, in degree mode.
- Angles of elevation and depression are measured from the horizontal and are equal for the same sight line.
- Law of Sines for AAS, ASA; Law of Cosines for SAS, SSS.



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