

## Reasoning and Proof: practice solutions, Grade 10 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

### 1 Continue the pattern



- a. Each term is 6 more than the previous one. Next terms:  $23 + 6 = 29$  and  $29 + 6 = 35$ .
- b. The 10th term is  $5 + 9 \cdot 6 = 59$ .
- c. It is inductive reasoning: we used the first four terms to guess a rule for all.

### 2 Hypothesis and conclusion



- a. Hypothesis: a whole number is divisible by 10. Conclusion: it ends in 0.
- b. Converse: "If a whole number ends in 0, then it is divisible by 10."
- c. Both are true: every multiple of 10 ends in 0, and every whole number ending in 0 is a multiple of 10 (for example  $370 = 10 \cdot 37$ ).

### 3 Inductive or deductive?



- a. Inductive (a pattern from specific cases).
- b. Deductive (a general fact applied to one case).
- c. Inductive (a generalization from measurements).
- d. Deductive (uses the definition of supplementary angles:  $180^\circ - 70^\circ = 110^\circ$ ).

### 4 Name the property



- a. Symmetric Property of Equality.
- b. Transitive Property of Equality.
- c. Reflexive Property of Congruence.
- d. Division Property of Equality (divide both sides by 3).
- e. Substitution Property of Equality.

### 5 Four related statements



Original: true (a square has four right angles).

Converse: "If a figure is a rectangle, then it is a square." False: a **2 in**  $\times$  **3 in** rectangle is not a square.

Inverse: "If a figure is not a square, then it is not a rectangle." False: the same **2**  $\times$  **3** rectangle is not a square but is a rectangle.

Contrapositive: "If a figure is not a rectangle, then it is not a square." True, like the original.

## 6 Biconditional or not?



- a. No. The converse is false:  $x = -4$  gives  $x^2 = 16$  too.
- b. Yes. The converse ("If an integer is divisible by 2, then it is even") is true, so: "An integer is even if and only if it is divisible by 2."
- c. Yes. A polygon is a triangle if and only if it has exactly three sides.

## 7 Justify each step



$5x - 9 = 26$  (Given).  $5x = 35$  (Addition Property of Equality: add 9).  $x = 7$  (Division Property of Equality: divide by 5).

Check:  $5 \cdot 7 - 9 = 26$ . So  $x = 7$ .

## 8 Find a counterexample



- a. The number 2 is prime and even.
- b. The number 6 is a multiple of 3 and is even (also 12, 18).

One counterexample is enough to show each statement false.

## 9 Trail midpoint



$M$  is the midpoint, so  $AM = MB$  (definition of midpoint). Then  $3x + 2 = 5x - 10$  (Substitution). Subtract  $3x$ :  $2 = 2x - 10$ . Add 10:  $12 = 2x$ . Divide by 2:  $x = 6$ .

$AM = 3 \cdot 6 + 2 = 20$  and  $MB = 5 \cdot 6 - 10 = 20$ . By the Segment Addition Postulate  $AB = 20 + 20 = 40$  m (about 131 ft).

### 10 Angle addition



Angle Addition Postulate:  $(2x + 10) + (3x - 5) = 125$ . So  $5x + 5 = 125$ ,  $5x = 120$ ,  $x = 24$ .

$m\angle ABD = 2 \cdot 24 + 10 = 58^\circ$  and  $m\angle DBC = 3 \cdot 24 - 5 = 67^\circ$ . Check:  $58 + 67 = 125$ .

### 11 Linear pair



Linear Pair Postulate: the angles are supplementary, so  $(4x + 12) + (2x + 6) = 180$ . Then  $6x + 18 = 180$ ,  $6x = 162$ ,  $x = 27$ .

$m\angle 1 = 4 \cdot 27 + 12 = 120^\circ$  and  $m\angle 2 = 2 \cdot 27 + 6 = 60^\circ$ . Check:  $120 + 60 = 180$ .

### 12 Reading vertical angles



$m\angle 3 = 85^\circ$ : vertical angles are congruent (Vertical Angles Theorem).

$m\angle 2 = 180 - 85 = 95^\circ$ :  $\angle 1$  and  $\angle 2$  are a linear pair.

$m\angle 4 = 95^\circ$ : vertical to  $\angle 2$ . Check:  $85 + 95 + 85 + 95 = 360$ .

### 13 Complete the proof



2. Distributive Property. 3. Subtraction Property of Equality (subtract  $2x$  from both sides). 4. Addition Property of Equality (add 6).

Check:  $3(11 - 2) = 27$  and  $2 \cdot 11 + 5 = 27$ .

#### 14 True or false?



- a. True. A conditional and its contrapositive are logically equivalent.
- b. False. “If a number is divisible by 6, then it is divisible by 3” is true, but its converse fails for 9.
- c. False. “If an animal is a dog, then it is a mammal” is true, but “If an animal is not a dog, then it is not a mammal” is false (a cat).

#### 15 Chain of congruent angles



$\angle A \cong \angle B$  and  $\angle B \cong \angle C$  give  $\angle A \cong \angle C$  (Transitive Property of Congruence). Using it again with  $\angle C \cong \angle D$  gives  $\angle A \cong \angle D$ . Congruent angles have equal measures, so  $m\angle D = 47^\circ$ .

#### 16 Rewrite as if-then



- a. “If an integer is divisible by 6, then it is divisible by 3.” Converse: “If an integer is divisible by 3, then it is divisible by 6.” False; 9 is a counterexample.
- b. “If a triangle has three congruent angles, then it is equiangular.” Converse: “If a triangle is equiangular, then it has three congruent angles.” True, so this is a definition and gives a biconditional.

#### 17 Congruent supplements



Since  $\angle 1$  and  $\angle 2$  are supplementary,  $m\angle 1 + m\angle 2 = 180^\circ$ . Since  $\angle 3$  and  $\angle 2$  are supplementary,  $m\angle 3 + m\angle 2 = 180^\circ$ . By substitution,  $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$ . Subtracting  $m\angle 2$  from both sides gives  $m\angle 1 = m\angle 3$ , so  $\angle 1 \cong \angle 3$  by the definition of congruent angles.

## 18 A prime pattern that fails



Try  $n = 11$ :  $11^2 - 11 + 11 = 121 = 11 \cdot 11$ , which is not prime. So  $n = 11$  is a counterexample and the conjecture is false.

Lesson: ten successes do not prove a general rule.

## 19 Midpoint converse



a. "If  $M$  is the midpoint of  $\overline{AB}$ , then  $M$  is on  $\overline{AB}$  and  $AM = MB$ " and "If  $M$  is on  $\overline{AB}$  and  $AM = MB$ , then  $M$  is the midpoint."

b.  $AM^2 = 2^2 + 3^2 = 13$  and  $MB^2 = (4 - 2)^2 + 3^2 = 13$ , so  $AM = MB = \sqrt{13}$ .

But  $M$  is not on  $\overline{AB}$  (its  $y$ -coordinate is 3), so it is not the midpoint. The hypothesis is true and the conclusion false: counterexample.

## 20 Chain of conditionals



Contrapositive of (2): if the lights do not go off, then the door is not locked. The lights are on, so the door is not locked. Contrapositive of (1): if the door is not locked, then the alarm did not ring. Therefore the alarm did not ring. This is deductive reasoning.

## 21 Full two-column proof



Statements and reasons: 1.  $B$  is between  $A$  and  $C$ ;  $AC = 24$  (Given). 2.  $AB + BC = AC$  (Segment Addition Postulate). 3.  $(3x - 1) + (x + 9) = 24$  (Substitution). 4.  $4x + 8 = 24$  (Simplify, combine like terms). 5.  $4x = 16$  (Subtraction Property of Equality). 6.  $x = 4$  (Division Property of Equality).

$AB = 3 \cdot 4 - 1 = 11$  ft and  $BC = 4 + 9 = 13$  ft;  $11 + 13 = 24$ .

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**Algebra with vertical angles**

Vertical angles are congruent:  $5x - 7 = 3x + 19$ . So  $2x = 26$  and  $x = 13$ .

$m\angle 1 = 5 \cdot 13 - 7 = 58^\circ$  (and  $m\angle 3 = 3 \cdot 13 + 19 = 58^\circ$ ).  $\angle 2$  is its linear pair, so

$m\angle 2 = 180 - 58 = 122^\circ$ .

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