

Reasoning and Proof: math lesson, Grade 10 – download the PDF

Geometry is not only about measuring shapes: it is about **knowing** why a fact is true. In this chapter you will learn how to spot patterns, write precise if-then statements, and build airtight arguments called proofs. By the end you will be able to defend a claim the way a mathematician does, one logical step at a time.

1. Inductive and deductive reasoning

D Inductive reasoning

Inductive reasoning means looking at several specific cases and forming a general guess, called a **conjecture**. A conjecture is only a prediction: it may turn out to be false.

D Deductive reasoning

Deductive reasoning means starting from facts that are already accepted (definitions, postulates, theorems, given information) and applying logic to reach a conclusion that **must** be true.

E**Example 1: finding and testing a conjecture**

Add consecutive odd numbers starting at 1: $1 = 1$, $1 + 3 = 4$, $1 + 3 + 5 = 9$, $1 + 3 + 5 + 7 = 16$. The totals **1, 4, 9, 16** are the perfect squares $1^2, 2^2, 3^2, 4^2$.

Conjecture: the sum of the first n odd numbers is n^2 . Checking $n = 5$:

$1 + 3 + 5 + 7 + 9 = 25 = 5^2$, which supports the conjecture but does not prove it.

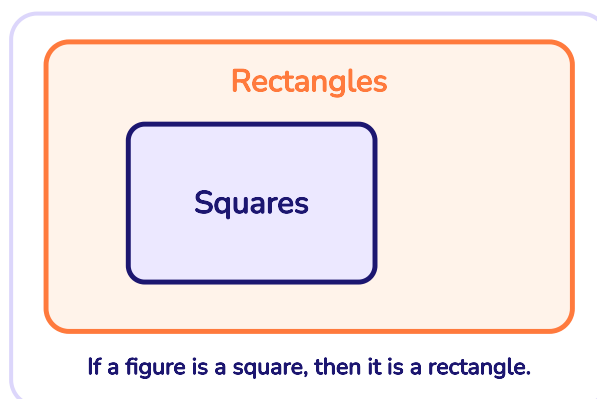
Now a deductive argument: all squares are rectangles; figure ***PQRS*** is a square; therefore ***PQRS*** is a rectangle. This conclusion is guaranteed.

**Careful**

Many correct examples never prove a general statement. A proof must work for *every* case.

2. Conditional statements

A **conditional statement** has the form “If p , then q .” The part after “if” is the **hypothesis** p ; the part after “then” is the **conclusion** q . We write it $p \rightarrow q$.



A conditional is **false** in exactly one situation: when the hypothesis is true and the conclusion is false. Such a case is called a counterexample (see Section 7). If the hypothesis is false, the conditional is considered true automatically.



Zyro's tip

On my planet we say: a promise is only broken when the condition happens and the result does not. "If it rains, I bring a coat" is not broken on a sunny day!

3. Converse, inverse, contrapositive and biconditionals

From one conditional you can build three related statements by swapping or negating the hypothesis and the conclusion.

Form	Pattern	Example
Conditional	If p , then q	If an animal is a dog, then it is a mammal. (true)
Converse	If q , then p	If an animal is a mammal, then it is a dog. (false)
Inverse	If not p , then not q	If an animal is not a dog, then it is not a mammal. (false)
Contrapositive	If not q , then not p	If an animal is not a mammal, then it is not a dog. (true)



Key fact

A conditional and its **contrapositive** always have the same truth value. The converse and the inverse also always have the same truth value as each other, but not necessarily the same as the original statement.

D**Biconditional**

When a conditional and its converse are both true, you can join them into a **biconditional**: “ p if and only if q ,” written $p \leftrightarrow q$. Definitions are always biconditionals.

E**Example 2: converse and biconditional**

Statement: “If a triangle is equilateral, then its three sides are congruent.” Converse: “If a triangle has three congruent sides, then it is equilateral.” Both are true, so we may write: “A triangle is equilateral **if and only if** its three sides are congruent.” By contrast, “If $x = 4$, then $x^2 = 16$ ” is true, but its converse “If $x^2 = 16$, then $x = 4$ ” is false because $x = -4$ also works, so no biconditional exists.

4. Properties of equality and congruence

A proof needs reasons. The most common reasons are the algebraic properties below. They also work for congruence of segments and angles.

Property	Statement
Addition / Subtraction	If $a = b$, then $a + c = b + c$ and $a - c = b - c$
Multiplication / Division	If $a = b$, then $ac = bc$ and, when $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$
Reflexive	$a = a$; $\overline{AB} \cong \overline{AB}$; $\angle A \cong \angle A$
Symmetric	If $a = b$, then $b = a$; if $\angle A \cong \angle B$, then $\angle B \cong \angle A$
Transitive	If $a = b$ and $b = c$, then $a = c$; the same for $\cong$
Substitution	If $a = b$, then b can replace a in any expression
Distributive	$a(b + c) = ab + ac$

Geometry adds a few more reasons you will use all the time: the **Segment Addition Postulate** (if B is between A and C , then $AB + BC = AC$), the **Angle Addition Postulate** (if ray BD lies inside $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$), the **Linear Pair Postulate** (angles of a linear pair are supplementary), and the **definition of midpoint** (a midpoint splits a segment into two congruent segments).

5. Two-column proofs

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How to write a two-column proof

1. Draw the figure and mark everything that is **Given**.
2. Write down what you must **Prove**.
3. Plan backward: what fact would give you the conclusion in one step?
4. Write numbered **Statements** on the left and a **Reason** for each on the right.
5. Each reason is a given, a definition, a postulate, a property, or an already proved theorem.

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Example 3: an algebra proof

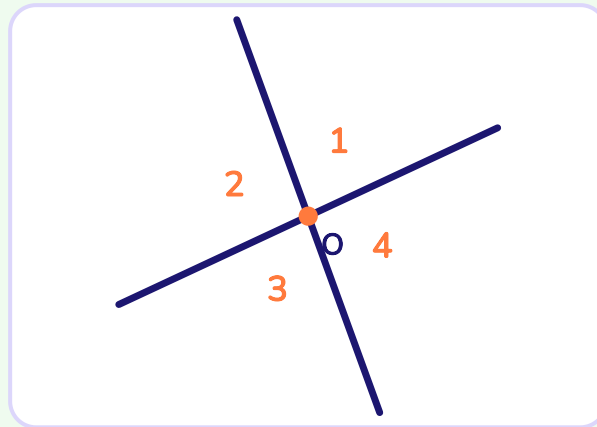
Given: $2x + 7 = 19$. Prove: $x = 6$.

Statements	Reasons
1. $2x + 7 = 19$	1. Given
2. $2x = 12$	2. Subtraction Property of Equality
3. $x = 6$	3. Division Property of Equality

Check: $2 \cdot 6 + 7 = 19$.

E**Example 4: the Vertical Angles Theorem**

In the figure, lines cross at O . Prove that $\angle 1 \cong \angle 3$.



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are a linear pair; $\angle 2$ and $\angle 3$ are a linear pair	1. Given (from the figure)
2. $m\angle 1 + m\angle 2 = 180^\circ$ and $m\angle 2 + m\angle 3 = 180^\circ$	2. Linear Pair Postulate
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	3. Substitution Property
4. $m\angle 1 = m\angle 3$	4. Subtraction Property of Equality
5. $\angle 1 \cong \angle 3$	5. Definition of congruent angles



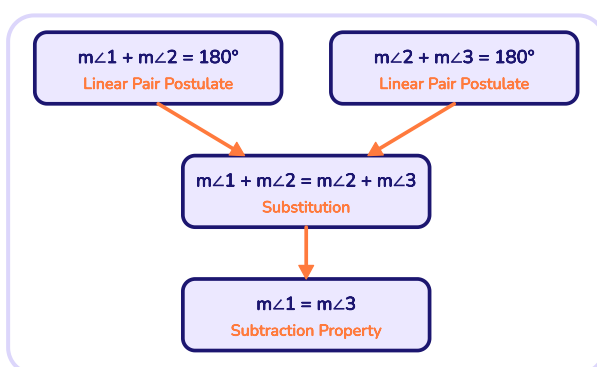
Careful

You cannot assume what a picture seems to show. Lengths, right angles and parallel lines must come from the given information or from a theorem.

6. Paragraph and flow proofs

The same logic can be displayed in other formats. A **paragraph proof** explains the argument in complete sentences. A **flow proof** puts each statement in a box and uses arrows to show which facts lead to which, with the reason written under each box.

Paragraph version of Example 4: Because $\angle 1$ and $\angle 2$ form a linear pair, $m\angle 1 + m\angle 2 = 180^\circ$. Because $\angle 2$ and $\angle 3$ also form a linear pair, $m\angle 2 + m\angle 3 = 180^\circ$. Substituting, $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$. Subtracting $m\angle 2$ from both sides gives $m\angle 1 = m\angle 3$, so the angles are congruent.



Choose the format that makes your reasoning clearest. All three formats (two-column, paragraph, flow) are equally valid when each claim has a reason.

7. Counterexamples

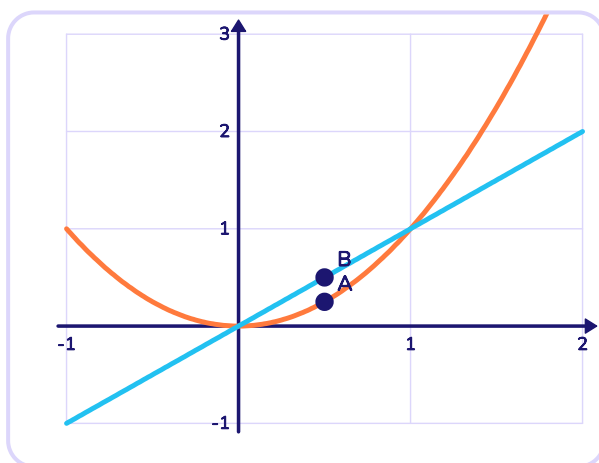
D Counterexample

A counterexample is one specific case where the hypothesis is true but the conclusion is false. A single counterexample is enough to disprove a general statement.

E Example 5: a pattern that breaks

For $n = 0, 1, 2, \dots, 39$, the value $n^2 + n + 41$ is always prime. Tempting conjecture: it is prime for every whole number n . But for $n = 40$:

$40^2 + 40 + 41 = 1600 + 81 = 1681 = 41 \cdot 41$. So $n = 40$ is a counterexample, and the conjecture is false. Another one: to disprove “If $x^2 = 25$, then $x = 5$,” take $x = -5$.



The graph shows a counterexample visually. The claim “ $x^2 > x$ for every number x ” fails at $x = 0.5$: point A has height 0.25 and point B has height 0.5 , so $x^2 < x$.

Key takeaways

- Inductive reasoning builds a conjecture from patterns; deductive reasoning proves a conclusion from accepted facts.
- “If p , then q ” is false only when p is true and q is false.
- The contrapositive always matches the original statement; the converse and inverse might not.
- A biconditional “ p if and only if q ” requires both the conditional and its converse to be true.
- Every step in a proof needs a reason: given, definition, postulate, property, or theorem.
- Two-column, paragraph and flow proofs are three ways to present the same logic.
- One counterexample disproves a statement; any number of examples never proves it.



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