

Ratios and Proportional Relationships: math lesson, Grade 7 – download the PDF

Prices per pound, speeds in miles per hour, recipes that you scale up for a crowd: all of them hide the same idea. In Grade 7 you learn to spot when two quantities grow together at a steady rate, to measure that rate even when it involves fractions, and to describe the whole relationship with one short equation, $y = kx$.

1. Ratios and rates

A **ratio** compares two quantities. We can write the ratio of 3 blue parts to 5 white parts as **3 : 5** or $\frac{3}{5}$. A **rate** is a ratio that compares two quantities with different units, such as 120 miles in 2 hours. A **unit rate** is a rate in which the second quantity is exactly 1, such as 60 miles per hour or \$2.50 per pound.

D Unit rate

If a quantity A goes with a quantity B , the unit rate is $\frac{A}{B}$: the amount of A for *one* unit of B .

Unit rates make comparing easy. A 12-ounce jar of salsa for \$3.60 costs \$0.30 per ounce. A 20-ounce jar for \$5.80 costs \$0.29 per ounce, so the larger jar is the better buy.

2. Unit rates with fractions

In real life the numbers are not always whole. A turtle might crawl a fraction of a mile in a fraction of an hour. The unit rate is still the first quantity divided by the second, and the result is a **complex fraction**: a fraction whose numerator, denominator, or both are fractions.

M Method: dividing fractions in a rate

1. Write the rate as $\frac{\text{first quantity}}{\text{second quantity}}$.
2. Rewrite the complex fraction as a division: $\frac{a}{b} \div \frac{c}{d}$.
3. Multiply by the reciprocal: $\frac{a}{b} \times \frac{d}{c}$.
4. Simplify and write the answer with its unit, such as "miles per hour".

E Example 1: a hiker's speed

A hiker walks $\frac{3}{4}$ mile in $\frac{1}{6}$ hour. Her unit rate is

$$\frac{3/4}{1/6} = \frac{3}{4} \times \frac{6}{1} = \frac{18}{4} = \frac{9}{2} = 4.5.$$

She walks 4.5 miles per hour.

blue: 3 parts = 15

white: 5 parts = 25

5	5	5	5	5	5	5	5
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total = 8 parts = 40

3. Proportional relationships in tables

Two quantities are in a **proportional relationship** when their ratio is the same for every pair of values. Doubling one quantity doubles the other, tripling one triples the other, and zero of one goes with zero of the other.

M Method: test a table

1. For every row, compute $\frac{y}{x}$.
2. If all the quotients are equal, the relationship is proportional. If even one is different, it is not.

E Example 2: a babysitting table

Is the relationship between hours worked and dollars earned proportional?

Hours (x)	Dollars (y)
2	17
3	25.5
5	42.5
8	68

The quotients are $\frac{17}{2} = 8.5$, $\frac{25.5}{3} = 8.5$, $\frac{42.5}{5} = 8.5$ and $\frac{68}{8} = 8.5$. They are all equal, so the relationship is proportional.

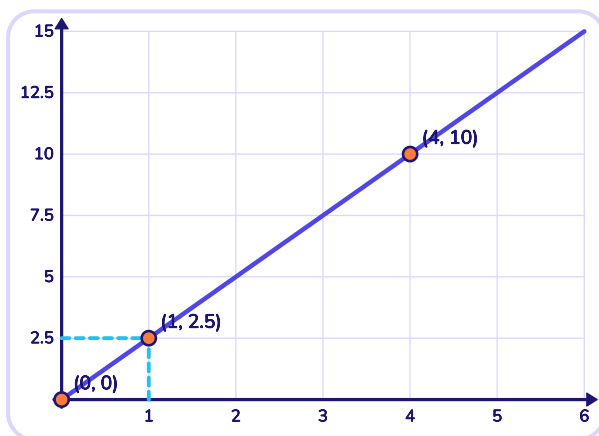


Common mistake

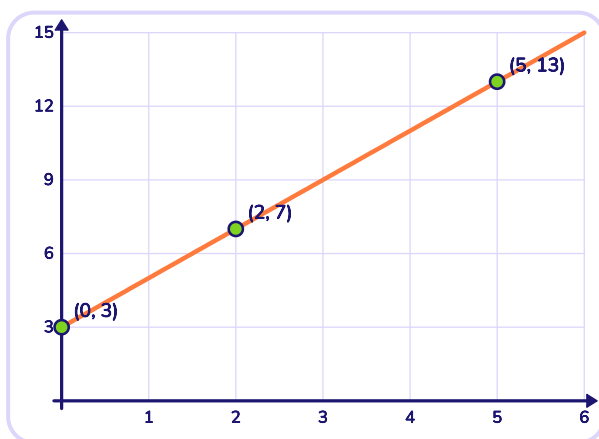
A table can go up by the same amount each time and still not be proportional. The values $(1, 4)$, $(2, 7)$, $(3, 10)$ increase by 3 each step, but the quotients $4, 3.5, 3.33 \dots$ are different. Always divide; never just look at the differences.

4. Proportional relationships in graphs

The graph of a proportional relationship has two features at once: it is a **straight line**, and the line passes through the **origin** $(0, 0)$. If either feature is missing, the relationship is not proportional. A straight line that crosses the y-axis anywhere except 0 is not enough.



Above, the line shows the cost of trail mix at \$2.50 per pound. Below, the line is straight but starts at 3 on the y-axis, so it is *not* proportional.



5. The constant of proportionality

D Constant of proportionality

In a proportional relationship, the number $k = \frac{y}{x}$ that is the same for every pair is called the **constant of proportionality**. It is the unit rate: the amount of y for one unit of x .

In the babysitting table, $k = 8.5$: you earn \$8.50 for each hour. In the trail mix graph, $k = 2.5$: each pound costs \$2.50.

6. Writing the equation $y = kx$

P Equation of a proportional relationship

If y is proportional to x with constant k , then $y = kx$. You can find any y by multiplying x by k , and any x by dividing y by k .

E Example 3: a faucet

A faucet fills a bucket at 2.4 gallons per minute. With x minutes and y gallons, the equation is $y = 2.4x$. To fill a 30-gallon tub, solve $2.4x = 30$, so $x = 30 \div 2.4 = 12.5$. It takes 12.5 minutes. In metric units, 2.4 gallons is about 9.1 liters, so the rate is about 9.1 liters per minute.

7. What the points $(0, 0)$ and $(1, r)$ mean

On the graph of $y = kx$, the point $(0, 0)$ says that zero of one quantity goes with zero of the other: zero pounds cost zero dollars. The point $(1, r)$ is the most useful point of all, because its y -

coordinate r is the unit rate. In the trail mix graph, $(1, 2.5)$ means that 1 pound costs \$2.50. Because $y = kx$, we always have $r = k$.

! Zyro's tip

On my planet we say: "Find the point with x equal to 1 and you have found the secret number." Divide any y by its x , and that secret number appears.

8. Multistep ratio problems

Many problems chain several steps: find a rate, then use it, then apply a percent. Draw a tape diagram, write each step on its own line, and check that the answer makes sense. Percent problems are ratio problems too: 25% off means you pay $100\% - 25\% = 75\%$ of the price, which is a multiplication by 0.75.

E Example 4: a sale and sales tax

A shirt costs \$48. It is on sale for 25% off, and then 8% sales tax is added.

Sale price: $48 \times 0.75 = 36$ dollars. With tax: $36 \times 1.08 = 38.88$ dollars. The shirt costs \$38.88.

E Example 5: sharing in a ratio

A bag has blue and white beads in the ratio $3 : 5$, with 40 beads in all. There are $3 + 5 = 8$ equal parts, and each part is $40 \div 8 = 5$ beads. So there are $3 \times 5 = 15$ blue beads and $5 \times 5 = 25$ white beads.

Key takeaways

- A unit rate compares a quantity to **one** unit of another. With fractions, divide by multiplying by the reciprocal.
- A relationship is proportional when $\frac{y}{x}$ is the same for every pair.
- The graph is a straight line through $(0, 0)$.
- The constant of proportionality k is the unit rate, and the equation is $y = kx$.
- The point $(1, r)$ shows the unit rate: $r = k$.
- For multistep problems, work one step at a time and use tape diagrams or percent multipliers.



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