

Ratios and Rate Reasoning: math lesson, Grade 6 – download the PDF

How many scoops of powder go in a smoothie, how fast does a bike go, which cereal box is the better deal? All of these questions are about comparing two quantities. In this chapter you will learn to describe, build and compare **ratios**, and to use **unit rates** to solve real problems.

1. What is a ratio?

D Definition: ratio

A **ratio** compares two quantities. The ratio of ***a*** to ***b*** can be written **in words** (***a* to *b***), **with a colon** (***a* : *b***), or **as a fraction** ($\frac{a}{b}$). The order matters: the first number goes with the first quantity named.

A bag holds 3 blue marbles and 2 green marbles. The ratio of blue to green marbles is **3 : 2**. The ratio of green to blue marbles is **2 : 3**, which is a different ratio. Look at the tape diagram below: each box is one marble.

Blue



Green



$$\text{Blue : Green} = 3 : 2$$

A ratio can compare **part to part** (blue to green, $3 : 2$) or **part to whole** (blue to all marbles, $3 : 5$), because there are $3 + 2 = 5$ marbles in all.

E

Example 1: writing ratios

A club has 8 sixth graders and 12 seventh graders.

- Sixth graders to seventh graders: $8 : 12$.
- Seventh graders to sixth graders: $12 : 8$.
- Sixth graders to all members: $8 : 20$, since $8 + 12 = 20$.



Watch the order

The ratio of 8 sixth graders to 12 seventh graders is $8 : 12$, not $12 : 8$. Always write the numbers in the same order as the words.

2. Equivalent ratios

D Definition: equivalent ratios

Two ratios are **equivalent** when they describe the same relationship. You get an equivalent ratio by **multiplying both numbers by the same nonzero number**, or by **dividing both by the same number**.

For example, **3 : 2** is equivalent to **6 : 4** (multiply by 2), to **9 : 6** (multiply by 3) and to **30 : 20** (multiply by 10). A ratio is in **simplest form** when the two numbers have no common factor other than 1: divide both by their greatest common factor (GCF).

E Example 2: simplifying and extending

Simplify **18 : 30**. The GCF of 18 and 30 is 6, so
 $18 : 30 = (18 \div 6) : (30 \div 6) = 3 : 5$.

Find the missing number in **3 : 5 = 21 : □**. Since **3 × 7 = 21**, multiply 5 by 7 as well:
5 × 7 = 35. So **3 : 5 = 21 : 35**.

⚠ Multiply, do not add

3 : 5 is *not* equivalent to **6 : 8**. Adding 3 to both numbers changes the relationship. Only multiplication or division keeps the ratio the same.

3. Ratio tables

A **ratio table** lists many equivalent ratios in columns. Every column is built from the first one by multiplying both rows by the same number.

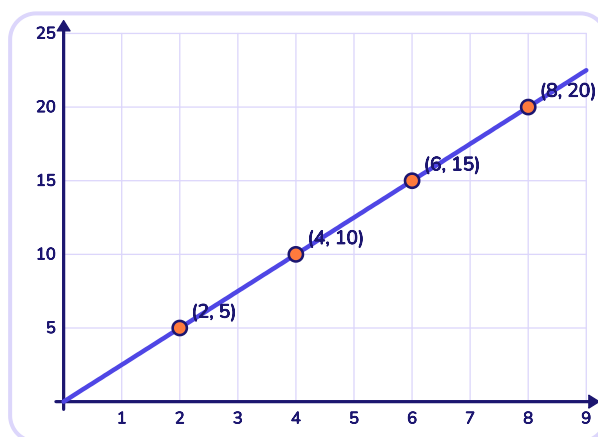
E Example 3: a lemonade recipe

The recipe uses 2 cups of juice for every 5 cups of water.

Juice (cups)	2	4	6	8	12
Water (cups)	5	10	15	20	30

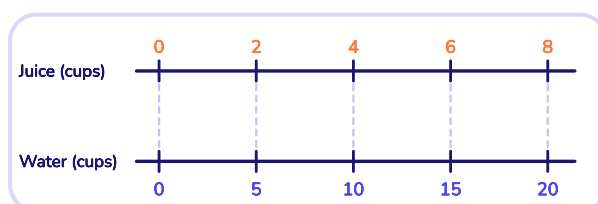
To use 12 cups of juice, notice that $12 = 2 \times 6$. Multiply the water by 6 too:
 $5 \times 6 = 30$. You need **30 cups of water**.

If you plot the pairs from the table as points, they line up on a straight line that passes through the origin. This is a signature of equivalent ratios.



4. Double number lines

A **double number line** shows two quantities on two parallel lines. Matching tick marks give equivalent ratios. It is a good way to see how both quantities grow together.



M**Method: solving with a double number line**

1. Draw two lines, one for each quantity, and mark 0 on both.
2. Place the known pair of values on matching marks.
3. Extend the marks by repeating the same jump on both lines.
4. Read the value that matches the one you need.

5. Unit rates**D****Definition: rate and unit rate**

A **rate** is a ratio of two quantities with **different units**, such as miles and hours. A **unit rate** compares a quantity to **1 unit** of the other quantity, such as 50 miles per 1 hour.

To find it, divide: $\frac{\text{first quantity}}{\text{second quantity}}$.

E**Example 4: finding a unit rate**

A hiker walks 9 miles in 4 hours. Her unit rate is $9 \div 4 = 2.25$ miles per hour. In 6 hours she walks $2.25 \times 6 = 13.5$ miles at the same pace.

A ratio $a : b$ gives two unit rates. For 9 miles in 4 hours you can also say $4 \div 9 \approx 0.44$ hour per mile. Choose the one that fits the question.

6. Unit price and speed

Unit price is the price of 1 unit (1 ounce, 1 pound, 1 item): $\text{unit price} = \frac{\text{price}}{\text{quantity}}$. **Speed** is the distance traveled per 1 unit of time: $\text{speed} = \frac{\text{distance}}{\text{time}}$. The lowest unit price is the best buy.

E**Example 5: which box is the better buy?**

Box A holds 16 oz of oatmeal for \$2.40. Box B holds 24 oz for \$3.30.

Box A: $2.40 \div 16 = 0.15$ dollar per ounce. Box B: $3.30 \div 24 = 0.1375$ dollar per ounce.

Box B costs less per ounce, so **Box B is the better buy.**

!**Zyro's tip**

On my planet we say: same unit, then compare! Turn each deal into “price per 1” and the winner appears right away. Units can be U.S. or metric: 1 mile is about 1.61 kilometers and 1 gallon is about 3.79 liters.

7. Comparing ratios

To compare two ratios, make them share a number. You can build equivalent ratios with the same second term, or compute a unit rate for each.

E**Example 6: which team is doing better?**

Team A has 6 wins for every 4 losses. Team B has 9 wins for every 7 losses.

Team A: $6 \div 4 = 1.5$ wins per loss. Team B: $9 \div 7 \approx 1.29$ wins per loss.

Team A has more wins per loss, so its record is better.

8. Solving ratio word problems with tape diagrams

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Method: split a total in a given ratio

1. Draw one box for each part of the ratio, in a separate row for each quantity.
2. Count all the boxes (the sum of the ratio numbers).
3. Divide the total by the number of boxes to get the value of one box.
4. Multiply by the number of boxes of each quantity, then check that the parts add up to the total.

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Example 7: sharing stickers

Ava and Ben share 35 stickers in the ratio **3 : 4**. There are **$3 + 4 = 7$** boxes, so each box is **$35 \div 7 = 5$** stickers. Ava gets **$3 \times 5 = 15$** stickers and Ben gets **$4 \times 5 = 20$** stickers. Check: **$15 + 20 = 35$** .

P

Property: equivalent ratios

If **$a : b$** is a ratio and **k** is a nonzero number, then **$a : b = (kimesa) : (kimesb)$** . Two ratios **$a : b$** and **$c : d$** are equivalent exactly when they have the same unit rate

$$\frac{a}{b} = \frac{c}{d}.$$

Key takeaways

- A ratio compares two quantities and is written $a : b$, a to b or $\frac{a}{b}$; the order matters.
- Equivalent ratios come from multiplying or dividing both numbers by the same number, never from adding.
- Ratio tables, double number lines and tape diagrams organize equivalent ratios.
- A unit rate compares a quantity to 1 unit: unit price = price $\div$ quantity, speed = distance $\div$ time.
- To compare ratios or deals, compute a unit rate for each and compare them.



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