

Integers and Rational Numbers: math lesson, Grade 6

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Temperatures below zero, diving depths, money you owe, floors in a parking garage under a building: all of these need numbers that go **below zero**. In this chapter you will meet negative numbers, learn to place them on a number line, compare them, and then use them to find points on a map called the coordinate plane.

1. Positive and negative numbers

A **positive number** is greater than zero. A **negative number** is less than zero and is written with a minus sign. Zero is neither positive nor negative. Negative numbers describe the opposite direction from a starting point: below sea level, a loss, a drop in temperature, a debt.

D Integers and rational numbers

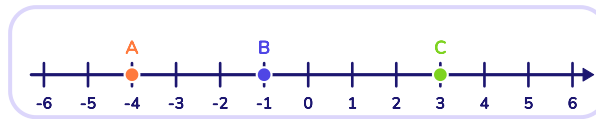
The **integers** are the whole numbers and their opposites:

$\dots, -3, -2, -1, 0, 1, 2, 3, \dots$ A **rational number** is any number that can be written as a fraction $\frac{a}{b}$ with a and b integers and $b \neq 0$. Fractions, mixed numbers, terminating decimals such as -2.75 and every integer are rational numbers.

Words help you choose the sign. “A gain of 8 yards” is $+8$. “A loss of 5 yards” is -5 . “12 feet below sea level” is -12 feet. “30 dollars owed” is -30 dollars.

2. The number line and opposites

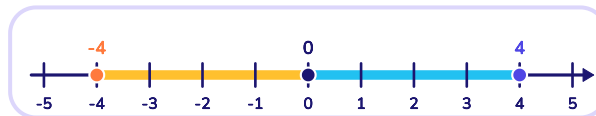
On a horizontal number line, positive numbers are to the right of zero and negative numbers are to the left. The farther right a point is, the greater the number.



In the figure, A is at -4 , B is at -1 and C is at 3 . The tick marks are one unit apart.

D Opposites

Two numbers are **opposites** when they are the same distance from zero but on different sides of zero. The opposite of 4 is -4 , and the opposite of -4 is 4 . The opposite of 0 is 0 . In symbols, $-(-a) = a$.



Opposites work for fractions and decimals too: the opposite of $\frac{3}{5}$ is $-\frac{3}{5}$, and the opposite of -6.2 is 6.2 .

E Example 1: finding opposites

What is the opposite of -17 ? The opposite of 8.5 ? The opposite of $-(-9)$?

The opposite of -17 is 17 . The opposite of 8.5 is -8.5 . Since $-(-9) = 9$, its opposite is -9 .

3. Absolute value

D Absolute value

The **absolute value** of a number is its distance from zero on the number line. We write $|x|$. A distance is never negative, so $|x| \geq 0$ for every number.

Because -4 and 4 are both four units from zero, $|-4| = 4$ and $|4| = 4$. Opposites always have the same absolute value. Also $|0| = 0$.

E Example 2: absolute values

Find $|-7.5|$, $|\frac{2}{3}|$ and $|-31|$.

$|-7.5| = 7.5$, $|\frac{2}{3}| = \frac{2}{3}$, and $|-31| = 31$. We just drop the sign and keep the distance.

Careful

Absolute value is not the same as “bigger.” The number -9 has a larger absolute value than 5 , but -9 is *less than* 5 . Absolute value tells you how far from zero, not which number is greater.

4. Comparing and ordering rational numbers

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Comparing with the number line

For any two numbers, the one farther to the right is greater. Therefore:

- every positive number is greater than every negative number;
- zero is greater than every negative number;
- between two negative numbers, the one closer to zero is greater, so $-2 > -8$.

M

Method: ordering mixed rational numbers

1. Sort the numbers by sign: negatives, zero, positives.
2. Write every number in the same form, usually decimals (or fractions with a common denominator).
3. For negatives, the larger the absolute value, the smaller the number.
4. Write the list from least to greatest, left to right on the number line.

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Example 3: ordering

Order from least to greatest: -2.5 , $\frac{1}{2}$, $-\frac{3}{4}$, 1.2 , -1 .

In decimals: -2.5 , 0.5 , -0.75 , 1.2 , -1 . The negatives are -2.5 , -1 and -0.75 ; the largest absolute value comes first. The order is

$$-2.5 < -1 < -\frac{3}{4} < \frac{1}{2} < 1.2.$$

5. Rational numbers in context

Choose the sign from the story, then compare as usual. A temperature of -9°F is colder than -4°F . A bank balance of -60 dollars means you owe more than at -35 dollars, so $-60 < -35$. A sea creature at -120 feet is deeper than one at -45 feet.



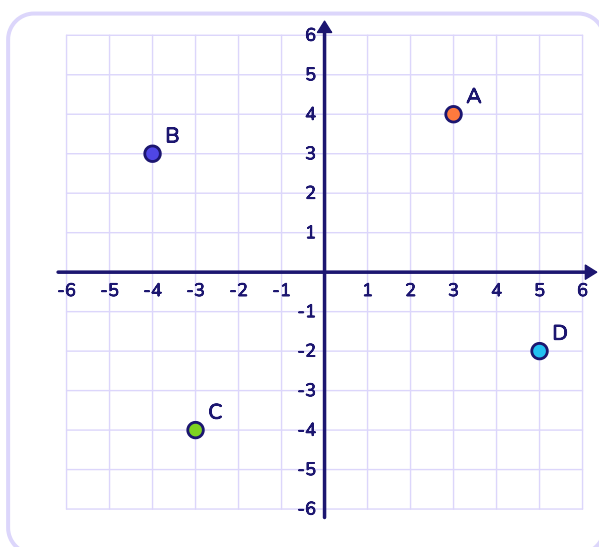
Zyro's tip

On my home planet we say: "Picture a thermometer." Higher on the thermometer means greater. A drop of 5 degrees from 2° lands at -3° , even though no one would call -3 "bigger" than 2!

To find how far apart two numbers are on a number line, think about the distance between them. Between -6 and 4 there are 6 units to reach zero and 4 more, so the distance is 10 .

6. The coordinate plane

A **coordinate plane** is made of two number lines that cross at right angles at the **origin** $(0, 0)$. The horizontal one is the **x-axis**; the vertical one is the **y-axis**. Every point has an **ordered pair** (x, y) : first move left or right by x , then up or down by y .



The axes cut the plane into four **quadrants**, numbered counterclockwise starting at the upper right.

Quadrant	Sign of x	Sign of y	Example
I	positive	positive	$(3, 4)$
II	negative	positive	$(-4, 3)$
III	negative	negative	$(-3, -4)$
IV	positive	negative	$(5, -2)$

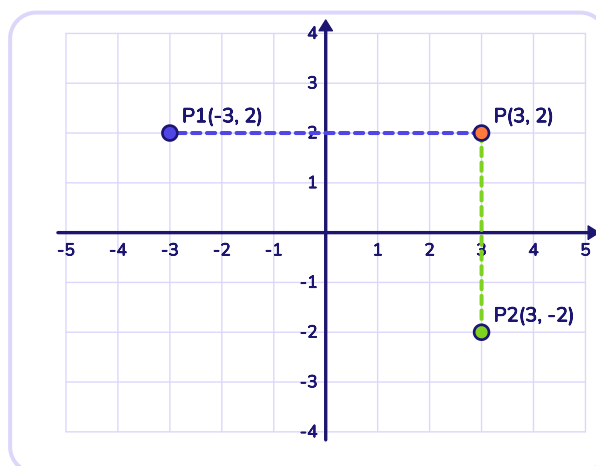
A point on an axis is in no quadrant. For example $(0, 5)$ lies on the y-axis.

7. Reflections across the axes

A **reflection** flips a point over an axis, like a mirror. The image is the same distance from the axis, on the other side.

P Reflection rules

- Across the **x-axis**: $(x, y) \rightarrow (x, -y)$. The y-coordinate becomes its opposite.
- Across the **y-axis**: $(x, y) \rightarrow (-x, y)$. The x-coordinate becomes its opposite.



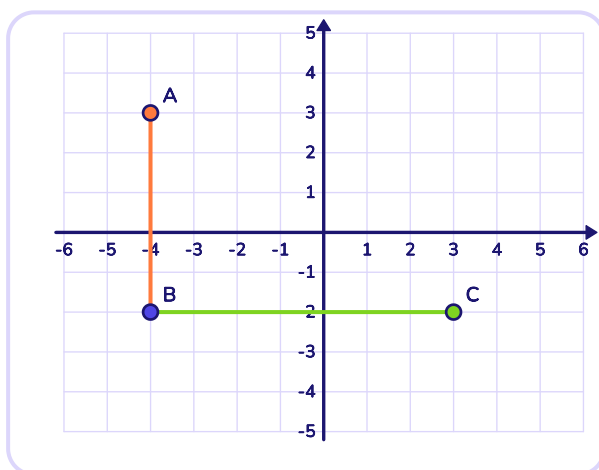
E Example 4: reflecting a point

Reflect $(-4, 3)$ across each axis.

Across the x-axis: $(-4, -3)$. Across the y-axis: $(4, 3)$. Reflecting across both axes gives $(4, -3)$: both coordinates are changed to their opposites.

8. Distance on the coordinate plane

When two points have the **same x-coordinate** they lie on a vertical line, and the distance is $|y_1 - y_2|$. When they have the **same y-coordinate** they lie on a horizontal line, and the distance is $|x_1 - x_2|$.



M Method: distance along a line

1. Check which coordinate is the same.
2. If the other coordinates have the same sign, subtract their absolute values.
3. If they have different signs, add their absolute values (you cross the axis).

E**Example 5: distances**

In the figure, $A(-4, 3)$ and $B(-4, -2)$ share $x = -4$. The y-coordinates have different signs, so $AB = 3 + 2 = 5$ units. $B(-4, -2)$ and $C(3, -2)$ share $y = -2$, so $BC = 4 + 3 = 7$ units.

Key takeaways

- Negative numbers are left of zero; opposites are the same distance from zero on opposite sides.
- $|x|$ is the distance from zero and is never negative.
- The number farther right is greater: $-8 < -2$, even though $|-8| > |-2|$.
- To order rational numbers, write them in the same form and use the number line.
- A coordinate plane has four quadrants: I $(+, +)$, II $(-, +)$, III $(-, -)$, IV $(+, -)$.
- Reflection across the x-axis changes the sign of y; across the y-axis it changes the sign of x.
- Distance on a horizontal or vertical line uses absolute values of the differing coordinates.

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