

# Rational and Irrational Numbers: practice solutions, Grade 8 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

## 1 Sort the numbers



Rational:  $5 = \frac{5}{1}$ ;  $-\frac{2}{3}$  is already a fraction;  $\sqrt{49} = 7$ ;  $0.\overline{3} = \frac{1}{3}$ ;  $2.75 = \frac{11}{4}$  (terminating decimal).

Irrational:  $\sqrt{11}$  because 11 is not a perfect square, and  $\pi$ , whose digits never end and never repeat.

## 2 Perfect squares and cubes



a) 9 because  $9^2 = 81$ . b) 12 because  $12^2 = 144$ . c)  $\frac{1}{2}$  because  $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ . d) 0.6 because  $0.6^2 = 0.36$ .

e) 3 because  $3^3 = 27$ . f) -4 because  $(-4)^3 = -64$ . g)  $\frac{2}{3}$  because  $\frac{2^3}{3^3} = \frac{8}{27}$ .

### 3 Fractions to decimals



- a)  $\frac{3}{8} = 0.375$ : terminates. b)  $\frac{5}{9} = 0.\overline{5}$ : repeats. c)  $\frac{7}{11} = 0.\overline{63}$ : repeats (the block **63**). d)  $\frac{1}{6} = 0.1\overline{6}$ : repeats (only the **6** repeats).

All four are rational numbers, as they come from fractions.

### 4 Between two integers



- a)  $16 < 20 < 25$ , so **4** and **5**. b)  $49 < 50 < 64$ , so **7** and **8**. c)  $81 < 90 < 100$ , so **9** and **10**.  
d)  $4 < 5 < 9$ , so **2** and **3**.  
e)  $27 < 30 < 64$ , so **3** and **4**. f)  $64 < 100 < 125$ , so **4** and **5**.

### 5 True or false?



- a) True:  $n = \frac{n}{1}$ . b) False:  $\sqrt{25} = 5$ , which is rational. c) False:  $\frac{22}{7} = 3.142857 \dots$  repeats, while  $\pi = 3.14159 \dots$  does not;  $\frac{22}{7}$  is only an approximation.  
d) True: for example  $0.62 = \frac{62}{100} = \frac{31}{50}$ . e) False: irrational means "not rational", so no number is both.

### 6 Solve the equations



- a)  $x = \pm 7$ . b)  $x = \pm 1.5$  because  $1.5^2 = 2.25$ . c)  $x = -2$ . d)  $x = \frac{1}{5}$ .  
e) **17** is not a perfect square, so  $x = \pm\sqrt{17}$ . f) **20** is not a perfect cube, so  $x = \sqrt[3]{20}$  (only one real solution).

## 7 Garden, box and tile



- a) Side =  $\sqrt{169} = 13$  ft, which is  $13 \times 0.3048 \approx 3.96$  m.
- b) Edge =  $\sqrt[3]{216} = 6$  in, since  $6^3 = 216$ .
- c) Side =  $\sqrt{50}$ . Since  $7.07^2 = 49.9849$  and  $7.08^2 = 50.1264$ , the side is about **7.07** cm.

## 8 Repeating decimals to fractions



- a)  $x = 0.2727\dots$ ,  $100x = 27.2727\dots$ , so  $99x = 27$  and  $x = \frac{27}{99} = \frac{3}{11}$ .
- b) Three digits repeat:  $1000x - x = 504$ , so  $x = \frac{504}{999} = \frac{56}{111}$ .
- c)  $10x - x = 14 - 1 = 13$ , so  $x = \frac{13}{9}$ .
- d)  $99x = 12$ , so  $x = \frac{12}{99} = \frac{4}{33}$ .

## 9 A digit in front of the block



- a)  $x = 0.2777\dots$ ;  $100x = 27.777\dots$  and  $10x = 2.777\dots$ . Subtract:  $90x = 25$ , so  $x = \frac{25}{90} = \frac{5}{18}$ .
- b)  $100x = 83.333\dots$ ,  $10x = 8.333\dots$ , so  $90x = 75$  and  $x = \frac{75}{90} = \frac{5}{6}$ .
- c)  $1000x = 145.4545\dots$ ,  $10x = 1.4545\dots$ , so  $990x = 144$  and  $x = \frac{144}{990} = \frac{8}{55}$ .

## 10 Squeeze the root



- a)  $3.6^2 = 12.96$  and  $3.7^2 = 13.69$ , so  $3.6 < \sqrt{13} < 3.7$ . Because **13** is much closer to 12.96,  $\sqrt{13} \approx 3.6$ .
- b)  $6.4^2 = 40.96$  and  $6.5^2 = 42.25$ , so  $6.4 < \sqrt{41} < 6.5$ , and  $\sqrt{41} \approx 6.4$ .
- c)  $8.66^2 = 74.9956$  and  $8.67^2 = 75.1689$ , so  $8.66 < \sqrt{75} < 8.67$ . **75** is much closer to 74.9956, so  $\sqrt{75} \approx 8.66$ .

### 11 Which is bigger?



- a)  $5.5^2 = 30.25 > 30$ , so  $\sqrt{30} < 5.5$ . b)  $\left(\frac{7}{2}\right)^2 = 12.25 > 12$ , so  $\sqrt{12} < \frac{7}{2}$ . c)  $2.1^3 = 9.261 > 9$ , so  $\sqrt[3]{9} < 2.1$ .  
d)  $\frac{22}{7} = 3.142857\dots$  and  $\pi = 3.14159\dots$ , so  $\pi < \frac{22}{7}$ .

### 12 Rational or irrational results



- a)  $3 + 4 = 7$ : rational. b)  $\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$ : rational, even though both factors are irrational. c)  $2\pi$ : irrational (nonzero rational times irrational).  
d)  $\sqrt{0.09} = 0.3$ : rational. e)  $4 + \sqrt{5}$ : irrational (rational plus irrational).

### 13 Name that point



Estimate each number.  $-\sqrt{3} \approx -1.73$  is negative, so it is *A*.  $\sqrt{2} \approx 1.41$  is *B*.  $\sqrt{5} \approx 2.24$  is *C*.  $\sqrt[3]{20} \approx 2.71$  (since  $2.7^3 = 19.683$  and  $2.8^3 = 21.952$ ) is *D*.  $\sqrt{12} \approx 3.46$  is *E*.

### 14 The aquarium



The volume is  $64 \times 1000 = 64,000 \text{ cm}^3$ . The edge is  $\sqrt[3]{64000} = 40$  cm since  $40^3 = 64,000$ .

In inches:  $40 \div 2.54 \approx 15.7$  in. The edge of the aquarium is 40 cm, about 15.7 inches.

### 15 Pattern or no pattern?



a) The number of zeros keeps growing, so no block repeats and the decimal never ends: irrational.

b) A block repeats, so it is rational; in fact it equals  $\frac{142857}{999999} = \frac{1}{7}$ .

c) The groups of 2s keep growing, so there is no repeating block: irrational.

d)  $3.14 = \frac{314}{100} = \frac{157}{50}$ : rational, a terminating decimal.

### 16 A tricky repeating decimal



Let  $x = 2.0363636 \dots$ . Then  $10x = 20.363636 \dots$  and  $1000x = 2036.3636 \dots$ . Subtract:  $990x = 2016$ , so  $x = \frac{2016}{990} = \frac{112}{55}$ .

Check:  $112 \div 55 = 2$  with remainder 2, and  $\frac{2}{55} = 0.0363636 \dots$ , so  $\frac{112}{55} = 2.\overline{036}$ .

### 17 Is $0.999 \dots$ equal to 1?



Let  $x = 0.999 \dots$ . Then  $10x = 9.999 \dots$ . Subtract:  $9x = 9$ , so  $x = 1$ . Therefore  $0.\overline{9} = 1$ : there is no gap between them, because the difference would have to be smaller than every positive number.

So  $0.\overline{9}$  is just another way to write  $1 = \frac{1}{1}$ , a rational number.

### 18 Order the negatives



Decimals:  $-\sqrt{7} \approx -2.646$  (since  $2.646^2 \approx 7.0013$ ),  $-2.6$ ,  $-\frac{5}{2} = -2.5$ ,  
 $-\sqrt[3]{15} \approx -2.466$  (since  $2.466^3 \approx 14.996$ ),  $-2.4$ .

The farther from zero, the smaller the negative number:

$$-\sqrt{7} < -2.6 < -\frac{5}{2} < -\sqrt[3]{15} < -2.4.$$

### 19 How many integers?



- a) Square everything:  $49 < n < 64$ , so  $n = 50, 51, \dots, 63$ . That is  $63 - 50 + 1 = 14$  integers; the smallest is 50 and the largest is 63.
- b)  $6.2^2 = 38.44$  and  $6.3^2 = 39.69$ , so  $38.44 < n < 39.69$ . Only  $n = 39$  works: exactly one integer.

### 20 The hypotenuse



- a) By the Pythagorean theorem,  $BC^2 = 9^2 + 5^2 = 81 + 25 = 106$ , so  $BC = \sqrt{106}$  ft.
- b)  $10.2^2 = 104.04$  and  $10.3^2 = 106.09$ , so  $10.2 < \sqrt{106} < 10.3$ , and the ramp is about 10.3 ft long.
- c) 106 is not a perfect square ( $10^2 = 100$  and  $11^2 = 121$ ), so  $\sqrt{106}$  is irrational.

### 21 Find the mistakes



- a)  $25^2 = 625$ , not 50. In fact  $7^2 = 49$  and  $8^2 = 64$ , so  $\sqrt{50} \approx 7.07$ .
- b) The fraction  $\frac{45}{100}$  is the terminating decimal 0.45. The repeating decimal is  $\frac{45}{99} = \frac{5}{11}$ .
- c)  $16 < 20 < 25$ , so  $\sqrt{20}$  is between 4 and 5.
- d) Cube roots of negative numbers exist:  $\sqrt[3]{-8} = -2$  because  $(-2)^3 = -8$ .



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