

Rational and Irrational Numbers: math lesson, Grade 8 – download the PDF

Every number you have met so far lives on a number line, but not every point on that line is a fraction. Some numbers, like $\sqrt{2}$ or π , have decimal digits that go on forever with no pattern at all. In this chapter you will sort numbers into **rational** and **irrational**, turn repeating decimals into fractions, work with square roots and cube roots, and learn how to **estimate, locate and compare** irrational numbers like a pro.

1. Rational numbers

D Rational number

A **rational number** is a number that can be written as a fraction $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

All of these are rational: $5 = \frac{5}{1}$, $-\frac{3}{4}$, $0.25 = \frac{1}{4}$, and $2\frac{1}{3} = \frac{7}{3}$. Every integer, every whole number and every fraction is rational.

When you divide the top of a fraction by the bottom, one of two things happens. The decimal either **terminates** (it stops), or it **repeats** (a block of digits comes back again and again). That is true for *every* rational number, because in long division there are only so many possible remainders, so a remainder must eventually come back.

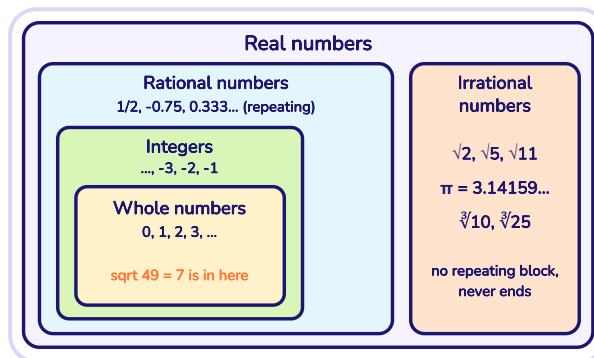
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Example 1

Write $\frac{7}{8}$ and $\frac{5}{11}$ as decimals.

$\frac{7}{8} = 0.875$, which terminates.

$\frac{5}{11} = 0.454545\dots = 0.\overline{45}$, which repeats. The bar goes over the block of digits that repeats.



2. Converting repeating decimals to fractions

Terminating decimals are easy: read the place value. For instance, $0.375 = \frac{375}{1000} = \frac{3}{8}$.

Repeating decimals need a clever trick that uses subtraction to make the endless tail disappear.

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Method: from repeating decimal to fraction

1. Name the number: x = the repeating decimal.
2. Multiply by **10** if one digit repeats, by **100** if two digits repeat, by **1000** if three repeat, and so on.
3. Subtract the original equation from the new one so the repeating tails cancel.
4. Solve for x and simplify the fraction.

E**Example 2**

Write $0.\overline{7}$ as a fraction.

Let $x = 0.777\dots$. Then $10x = 7.777\dots$. Subtract: $10x - x = 7.777\dots - 0.777\dots$,
so $9x = 7$ and $x = \frac{7}{9}$.

E**Example 3**

Write $0.\overline{36}$ as a fraction.

Two digits repeat, so multiply by 100. Let $x = 0.3636\dots$. Then $100x = 36.3636\dots$.
Subtract: $99x = 36$, so $x = \frac{36}{99} = \frac{4}{11}$.

E**Example 4**

Write $0.4\overline{5} = 0.45555\dots$ as a fraction.

Here only the 5 repeats, but a 4 sits in front of it. Make two equations with the same repeating tail: $100x = 45.555\dots$ and $10x = 4.555\dots$. Subtract: $90x = 41$, so
 $x = \frac{41}{90}$.



Watch out

$0.\overline{45}$ is **not** $\frac{45}{100}$. A repeating decimal never equals the fraction you get by just reading the digits. Use the subtraction method: $0.\overline{45} = \frac{45}{99} = \frac{5}{11}$.

3. Square roots and cube roots



Square root and cube root

The **square root** $\sqrt{p}$ of a positive number p is the positive number whose square is p .

The **cube root** $\sqrt[3]{p}$ is the number whose cube is p .

For example, $\sqrt{81} = 9$ because $9^2 = 81$, and $\sqrt[3]{125} = 5$ because $5^3 = 125$. Numbers like **81** and **125** are called **perfect squares** and **perfect cubes**. Memorize these:

n	1	2	3	4	5	6	7	8	9	10	11	12
n^2	1	4	9	16	25	36	49	64	81	100	121	144
n^3	1	8	27	64	125	216	343	512	729	1000		

Two things to remember. First, the equation $x^2 = p$ has **two** solutions when $p > 0$: $x = \sqrt{p}$ and $x = -\sqrt{p}$. Second, a cube root can be negative: $\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$. A square root of a negative number does not exist among real numbers, since no real number squared gives a negative result.

E**Example 5**

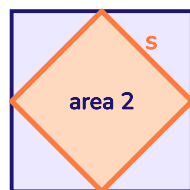
Solve $x^2 = \frac{64}{81}$ and $x^3 = -\frac{1}{27}$.

$$x = \pm\sqrt{\frac{64}{81}} = \pm\frac{8}{9}. \text{ For the cube, } x = \sqrt[3]{-\frac{1}{27}} = -\frac{1}{3}.$$

4. Irrational numbers**D****Irrational number**

An **irrational number** is a real number that is **not** rational. It cannot be written as a fraction of two integers. Its decimal expansion never ends and never repeats.

If a positive integer is not a perfect square, its square root is irrational. So $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{11}$ are irrational, while $\sqrt{49} = 7$ is rational. In the same way $\sqrt[3]{10}$ is irrational because 10 is not a perfect cube. The number π , the circumference of a circle divided by its diameter, is irrational too.



Big square: $2 \times 2 = 4$

Four corner triangles:
 $4 \times (1/2) = 2$

Tilted square: $4 - 2 = 2$

So $s^2 = 2$, and $s = \sqrt{2}$

The picture shows where $\sqrt{2}$ comes from: a tilted square inside a 2×2 square has area $4 - 4 \times \frac{1}{2} = 2$. Its side s satisfies $s^2 = 2$, so $s = \sqrt{2} = 1.41421356 \dots$ A real length, but not a fraction.



Useful fact

The sum of a rational number and an irrational number is irrational, and so is the product of a nonzero rational number and an irrational number. For instance, $3 + \sqrt{5}$ and 2π are irrational.



Common mix-up

3.14 and $\frac{22}{7}$ are only **rational approximations** of π . The true value $\pi = 3.14159265\dots$ never stops and never repeats. Also, a long decimal is not automatically irrational: $0.\overline{142857} = \frac{1}{7}$ is rational.

5. Estimating irrational numbers

You do not need a calculator to find where an irrational number sits. Trap it between two perfect squares (or cubes), then squeeze it with decimals.



Method: estimate $\sqrt{n}$

1. Find the two consecutive perfect squares around n .
2. Take their square roots to get two consecutive integers.
3. Test decimals between them by squaring, until you reach the accuracy you need.

E**Example 6**

Estimate $\sqrt{30}$ to the nearest hundredth.

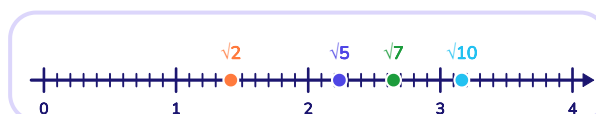
$25 < 30 < 36$, so $5 < \sqrt{30} < 6$. Test tenths: $5.4^2 = 29.16$ and $5.5^2 = 30.25$, so $5.4 < \sqrt{30} < 5.5$. Test hundredths: $5.47^2 = 29.9209$ and $5.48^2 = 30.0304$, so $5.47 < \sqrt{30} < 5.48$. Since 30 is much closer to 30.0304 than to 29.9209 , $\sqrt{30} \approx 5.48$.

!**Zyro's tip**

On my home planet we always ask: is n closer to the lower or the higher perfect square? 30 is 5 away from 25 and 6 away from 36 , so $\sqrt{30}$ is a bit closer to 5 than to 6 .

6. Locating irrational numbers on a number line

Once you have a good estimate, you can place the number on a number line. Here are four square roots located with tenths marked between the integers.

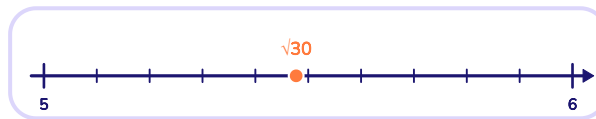


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Example 7

Locate $\sqrt{7}$ and $-\sqrt{7}$.

$4 < 7 < 9$, so $2 < \sqrt{7} < 3$. Since $2.6^2 = 6.76$ and $2.7^2 = 7.29$, we get $2.6 < \sqrt{7} < 2.7$: plot it between the marks 2.6 and 2.7. The opposite $-\sqrt{7}$ is the mirror image on the other side of 0, between -2.7 and -2.6 .



7. Comparing and ordering real numbers

The **real numbers** are all the rational and irrational numbers together: every point of the number line. To compare two of them, you have two reliable tools.

- **Decimals:** write each number as a decimal with enough digits to see which is bigger.
- **Squares or cubes:** for positive numbers, $a < b$ exactly when $a^2 < b^2$. So you can compare $\sqrt{21}$ with 4.6 by comparing 21 with $4.6^2 = 21.16$.

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Example 8

Order from least to greatest: $\sqrt{5}$, 2.3, $\frac{9}{4}$, $\sqrt[3]{12}$.

Decimals: $\sqrt{5} \approx 2.236$, 2.3 , $\frac{9}{4} = 2.25$, $\sqrt[3]{12} \approx 2.289$ (because $2.29^3 \approx 12.009$). The order is

$$\sqrt{5} < \frac{9}{4} < \sqrt[3]{12} < 2.3.$$



Negative numbers

The order flips for negatives: $-\sqrt{10} < -3$ because $\sqrt{10} \approx 3.16$ is farther from 0 than 3 is.

Key takeaways

- A rational number is a fraction $\frac{a}{b}$ of integers ($b \neq 0$); its decimal terminates or repeats.
- To convert a repeating decimal: multiply by 10, 100, 1000... to line up the tails, subtract, and solve. For example $0.\overline{36} = \frac{4}{11}$.
- $\sqrt{p}$ is the positive number whose square is p ; $x^2 = p$ has two solutions $\pm\sqrt{p}$. A cube root can be negative.
- If n is a positive integer that is not a perfect square, $\sqrt{n}$ is irrational. π is irrational.
- Estimate an irrational number by trapping it between consecutive perfect squares or cubes, then squeezing it with tenths and hundredths.
- Rational and irrational numbers together make the real numbers, and each one has its place on the number line.



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