

Radicals and Exponential Functions: math lesson, Grade 9 – download the PDF

Some numbers never end, like $\sqrt{2}$, and some quantities explode, like a rumor that doubles every hour. In this chapter you will learn two powerful tools. First, radicals: how to simplify them, combine them and clean up fractions that have a root in the denominator. Second, exponential functions: the equations behind growing populations, shrinking values and bank accounts that earn interest on interest.

1. Square roots and perfect squares

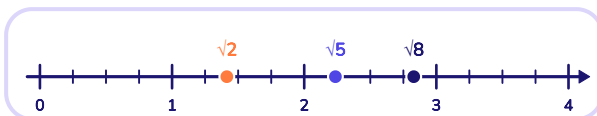
D Square root

The principal square root of a number $a \geq 0$, written $\sqrt{a}$, is the nonnegative number whose square is a . So $\sqrt{49} = 7$ because $7^2 = 49$. The symbol $\sqrt{\quad}$ is called a radical and the number under it is the radicand.

A **perfect square** is the square of a whole number. Knowing the first twelve by heart makes every radical much easier.

n	1	2	3	4	5	6	7	8	9	10	11	12
n^2	1	4	9	16	25	36	49	64	81	100	121	144

When the radicand is not a perfect square, the root is irrational: its decimal never ends and never repeats. We keep it as a radical for an exact answer and use a calculator only when we need an approximation.



2. Simplifying square roots

P Product rule for radicals

For $a \geq 0$ and $b \geq 0$: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$. In the same way, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ when $b > 0$.

M How to simplify $\sqrt{n}$

1. Find the largest perfect square that divides n .
2. Write n as that perfect square times another factor.
3. Split the root with the product rule and take the square root of the perfect square.

E Example 1

Simplify $\sqrt{72}$. The largest perfect square factor is 36, since $72 = 36 \cdot 2$. Then $\sqrt{72} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$.

E**Example 2**

Simplify $\sqrt{12x^2}$ for $x \geq 0$. We have $12x^2 = 4 \cdot 3 \cdot x^2$, so
 $\sqrt{12x^2} = 2 \cdot \sqrt{3} \cdot x = 2x\sqrt{3}$.

**Common mistake**

The root of a sum is *not* the sum of the roots. $\sqrt{9 + 16} = \sqrt{25} = 5$, but
 $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$. The product rule works for multiplication only.

3. Operations with radicals

Radicals behave like variables. You can add or subtract **like radicals**, meaning radicals with the same radicand, by adding their coefficients: $3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5}$. Radicals with different radicands cannot be combined, so $3\sqrt{2} + 2\sqrt{3}$ stays as it is. Always simplify first, because hidden like radicals often appear.

To multiply, multiply the coefficients together and the radicands together: $a\sqrt{b} \cdot c\sqrt{d} = ac\sqrt{bd}$. To expand products of sums, use the distributive property or FOIL, exactly as with polynomials.

E**Example 3**

Compute $\sqrt{12} + \sqrt{27}$. Simplify each root: $\sqrt{12} = 2\sqrt{3}$ and $\sqrt{27} = 3\sqrt{3}$. Then the sum is $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$.

E**Example 4**

Expand $(3 + \sqrt{2})^2$. Using $(a + b)^2 = a^2 + 2ab + b^2$: $9 + 6\sqrt{2} + 2 = 11 + 6\sqrt{2}$.

4. Rationalizing denominators

A fraction is considered fully simplified when its denominator contains no radical. To **rationalize** a denominator, multiply the numerator and the denominator by the same number so that the denominator becomes a whole number. The value of the fraction does not change because you are multiplying by 1.

M**Two cases**

- One radical in the denominator, such as $\frac{6}{\sqrt{3}}$: multiply top and bottom by $\sqrt{3}$.
- A sum or difference, such as $\frac{4}{3 + \sqrt{5}}$: multiply top and bottom by the **conjugate** $3 - \sqrt{5}$. Since $(a + b)(a - b) = a^2 - b^2$, the radical disappears.

E**Example 5**

$$\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}.$$

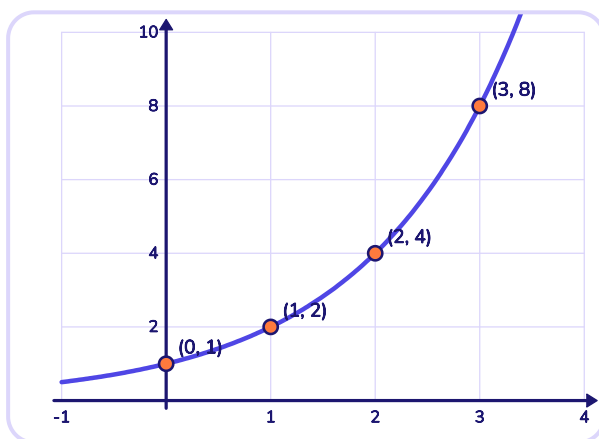
$$\frac{4}{3 + \sqrt{5}} = \frac{4(3 - \sqrt{5})}{(3 + \sqrt{5})(3 - \sqrt{5})} = \frac{4(3 - \sqrt{5})}{9 - 5} = 3 - \sqrt{5}.$$

5. Exponential growth

D Exponential function

An exponential function has the form $f(x) = a \cdot b^x$, where $a \neq 0$ is the initial value (the value at $x = 0$), and $b > 0, b \neq 1$, is the growth factor. The variable is in the exponent, which is what makes it different from a polynomial.

When $b > 1$ the function models **exponential growth**. If a quantity grows by a percent rate r (written as a decimal) each period, the growth factor is $b = 1 + r$. A 4% increase means $b = 1.04$.



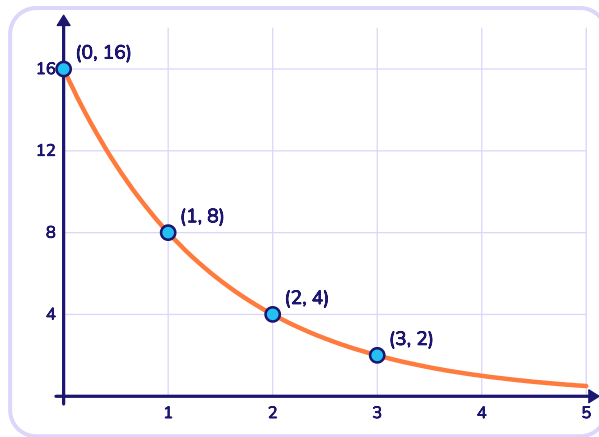
E Example 6

A town has 12,000 residents and grows 4% each year. The model is

$P(t) = 12,000 \cdot 1.04^t$. After 3 years, $P(3) = 12,000 \cdot 1.04^3 \approx 13,498$ residents. A bacteria colony of 200 cells that doubles every hour follows $N(t) = 200 \cdot 2^t$, so after 5 hours $N(5) = 200 \cdot 32 = 6,400$.

6. Exponential decay

When $b < 1$ the function models **exponential decay**. If a quantity loses a percent rate r each period, then $b = 1 - r$. A 15% loss gives $b = 0.85$. The graph falls quickly at first, then flattens and gets closer and closer to the x-axis without ever touching it.



E Example 7

A new car costs \$24,000 and loses 15% of its value every year. Its value after t years is $V(t) = 24,000 \cdot 0.85^t$. After 3 years, $V(3) = 24,000 \cdot 0.614125 = \$14,739$.

7. Compound interest

When a bank adds interest to your balance and then pays interest on the new, larger balance, the interest is **compounded**. This is exponential growth.

P Compound interest formula

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

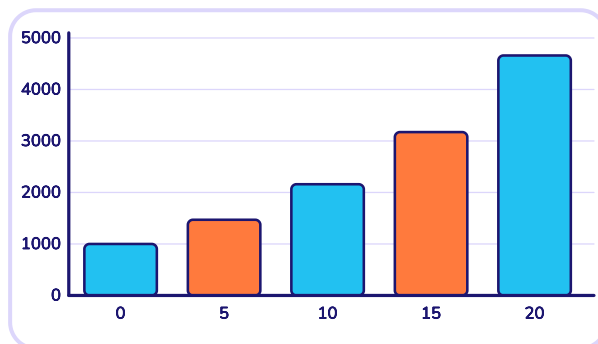
Here P is the principal (starting amount), r the annual rate as a decimal, n the number of compounding periods per year, t the time in years and A the final balance.

E Example 8

You deposit \$2,000 at 6% compounded annually ($n = 1$) for 4 years:

$A = 2000 \cdot 1.06^4 \approx \$2,524.95$. Now take \$1,500 at 8% compounded quarterly (

$n = 4$) for 2 years: $A = 1500 \left(1 + \frac{0.08}{4}\right)^8 = 1500 \cdot 1.02^8 \approx \$1,757.49$.

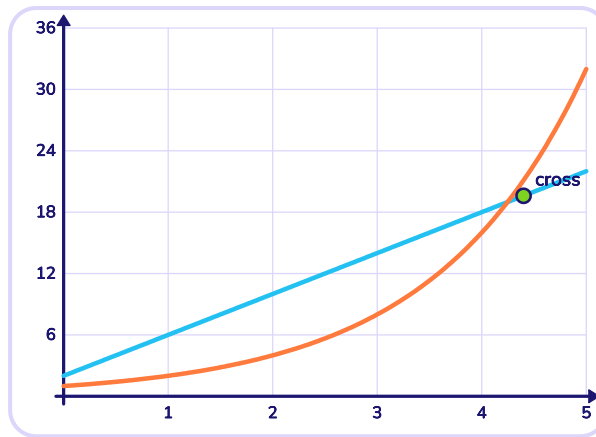


The more often interest is compounded, the more you earn, but the gain from monthly instead of yearly compounding is modest. Time matters much more: look at how the bars above grow faster and faster.

8. Comparing linear and exponential models

A linear function adds the **same amount** each time x increases by 1 (constant difference, the slope). An exponential function multiplies by the **same factor** each time (constant ratio). Compare $f(x) = 4x + 2$ and $g(x) = 2^x$.

x	0	1	2	3	4	5
$f(x) = 4x + 2$	2	6	10	14	18	22
$g(x) = 2^x$	1	2	4	8	16	32



At first the line is above the curve, but around $x \approx 4.4$ the exponential overtakes it and never looks back. **Any** exponential function with $a > 0$ and $b > 1$ eventually passes any linear function, no matter how steep the line is.

! Zyro's tip

To identify a model from a table, compute differences and ratios of consecutive outputs. Equal differences mean linear, equal ratios mean exponential. On my home planet, we check both before trusting any prediction!

9. Geometric sequences

D Geometric sequence

A sequence is geometric if each term is the previous term multiplied by a constant r , the common ratio. The explicit formula is $a_n = a_1 \cdot r^{n-1}$.

A geometric sequence is an exponential function whose inputs are the whole numbers $1, 2, 3, \dots$. To find r , divide any term by the one before it.

E

Example 9

For 3, 6, 12, 24, ... the ratio is $r = 2$, so $a_n = 3 \cdot 2^{n-1}$ and $a_8 = 3 \cdot 2^7 = 384$. For 81, 27, 9, ... the ratio is $r = \frac{1}{3}$, so $a_6 = 81 \cdot \left(\frac{1}{3}\right)^5 = \frac{1}{3}$.

Key takeaways

- $\sqrt{ab} = \sqrt{a}\sqrt{b}$ for $a, b \geq 0$; simplify by pulling out the largest perfect square factor. Never split a sum under a root.
- Add or subtract only like radicals; multiply coefficients and radicands separately.
- Rationalize a denominator by multiplying by the radical itself, or by the conjugate when there is a sum or difference.
- $f(x) = a \cdot b^x$: growth if $b > 1$ ($b = 1 + r$), decay if $0 < b < 1$.
- Compound interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$.
- Linear means constant difference, exponential means constant ratio; a growing exponential always wins in the long run.
- Geometric sequence: $a_n = a_1 \cdot r^{n-1}$.



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