

Quadrilaterals and Polygons: math lesson, Grade 10

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Look at a tile floor, a stop sign, a kite in the sky, or a phone screen. Polygons are everywhere, and four-sided ones, the quadrilaterals, come in a whole family with strict rules. In this chapter you will learn to compute angles in any polygon, recognize each quadrilateral from its properties, prove that a figure belongs to a family, and even do it with coordinates.

1. Polygons and their angle sums

D Polygon

A **polygon** is a closed figure made of straight segments (its sides) that meet only at their endpoints (its vertices). A polygon with n sides is an **n -gon**. It is **convex** if every interior angle is less than 180° , and **regular** if all its sides and all its angles are congruent.

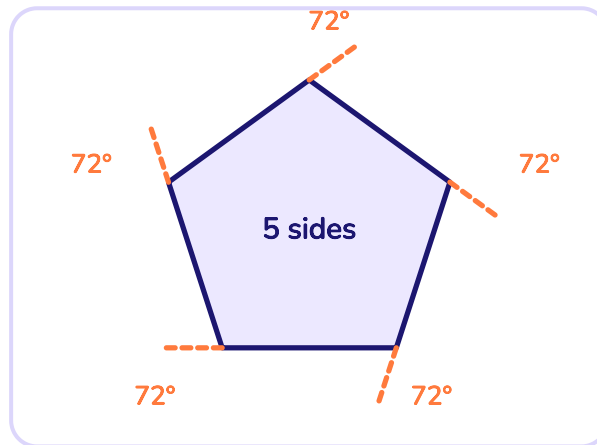
Pick one vertex of an n -gon and draw all the diagonals from it. You cut the polygon into $n - 2$ triangles, and each triangle contributes 180° .

P**Angle sum theorems (convex polygon with n sides)**

Sum of the interior angles: $S = (n - 2) \cdot 180^\circ$.

Sum of the exterior angles (one at each vertex): 360° .

Regular polygon: each interior angle is $\frac{(n - 2) \cdot 180^\circ}{n}$ and each exterior angle is $\frac{360^\circ}{n}$.



At each vertex, an interior angle and its exterior angle are a linear pair, so they add up to 180° . A quadrilateral has $n = 4$, so its angles always add up to $2 \cdot 180^\circ = 360^\circ$.

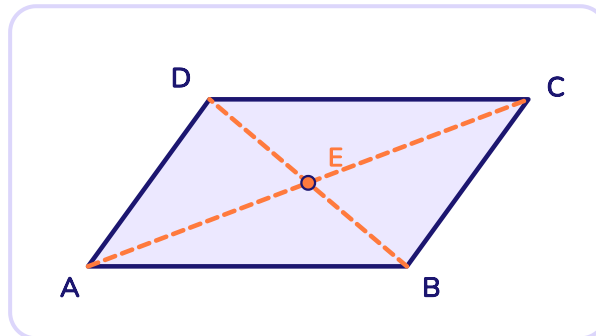
E**Example 1: a regular 12-gon**

Interior sum: $(12 - 2) \cdot 180^\circ = 1800^\circ$. Each interior angle: $1800^\circ \div 12 = 150^\circ$. Each exterior angle: $360^\circ \div 12 = 30^\circ$, and indeed $150^\circ + 30^\circ = 180^\circ$.

2. Properties of parallelograms

D Parallelogram

A **parallelogram** is a quadrilateral whose two pairs of opposite sides are parallel.



P Parallelogram theorems

- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary (they add up to 180°).
- The diagonals bisect each other: they meet at their common midpoint.

E Example 2: finding angles

In parallelogram $ABCD$, $\angle A = (4x + 10)^\circ$ and $\angle B = (6x - 10)^\circ$. Angles A and B are consecutive, so $4x + 10 + 6x - 10 = 180$, which gives $10x = 180$ and $x = 18$. Then $\angle A = 82^\circ$ and $\angle B = 98^\circ$, and $\angle C = 82^\circ$, $\angle D = 98^\circ$.

3. Proving that a quadrilateral is a parallelogram

The theorems above can be turned around. If you can check just one of the following conditions in a quadrilateral, it is automatically a parallelogram.

P Ways to prove a parallelogram

- a. Both pairs of opposite sides are parallel (the definition).
- b. Both pairs of opposite sides are congruent.
- c. Both pairs of opposite angles are congruent.
- d. The diagonals bisect each other.
- e. One pair of opposite sides is both parallel and congruent.

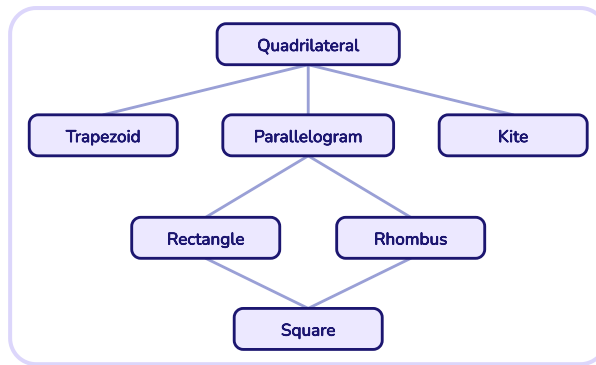
⚠ Careful

One pair of congruent sides is not enough, and one pair of parallel sides is not enough either. An isosceles trapezoid has congruent legs, and every trapezoid has parallel bases, yet neither is a parallelogram. Condition (e) needs *both* properties on the *same* pair of sides.

4. Rectangles, rhombuses, and squares

D Special parallelograms

- A **rectangle** is a quadrilateral with four right angles.
- A **rhombus** is a quadrilateral with four congruent sides.
- A **square** is a quadrilateral with four right angles and four congruent sides.



All three are parallelograms, so they inherit every parallelogram property. They also have extra ones.

P Extra properties

- Rectangle: the diagonals are congruent. (Conversely, a parallelogram with congruent diagonals is a rectangle.)
- Rhombus: the diagonals are perpendicular, and each diagonal bisects a pair of opposite angles. (A parallelogram with perpendicular diagonals is a rhombus.)
- Square: all the properties of both a rectangle and a rhombus.

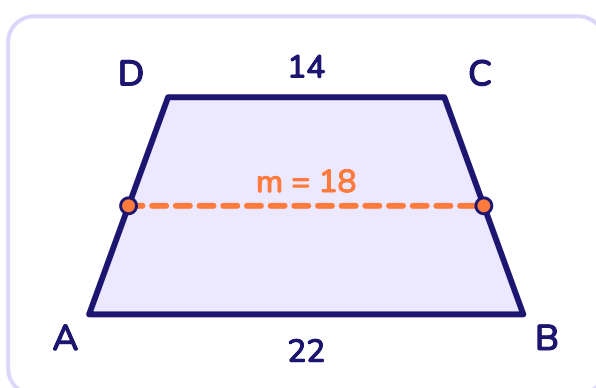
E Example 3: a rhombus from its diagonals

A rhombus has diagonals of **16 cm** and **30 cm**. They are perpendicular and bisect each other, so each side is the hypotenuse of a right triangle with legs **8** and **15**. Side $= \sqrt{8^2 + 15^2} = \sqrt{289} = 17 \text{ cm}$. The perimeter is $4 \cdot 17 = 68 \text{ cm}$ and the area is $\frac{16 \cdot 30}{2} = 240 \text{ cm}^2$.

5. Trapezoids and kites

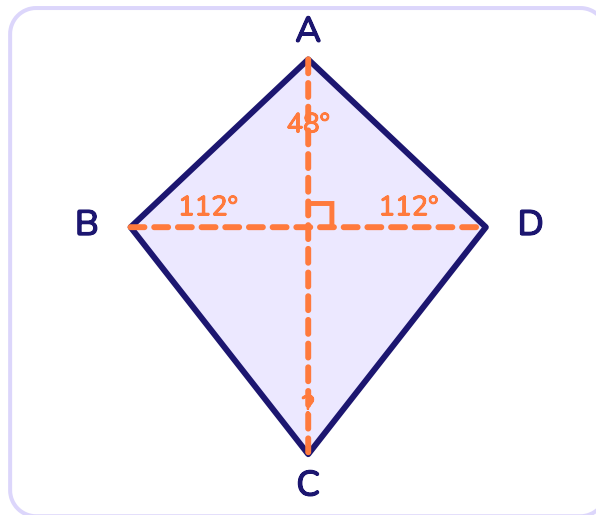
D Trapezoid and kite

In this book, a **trapezoid** has exactly one pair of parallel sides (the **bases**); the other two sides are the **legs**. A trapezoid is **isosceles** if its legs are congruent. A **kite** has two pairs of congruent adjacent sides, and its opposite sides are not both congruent unless it is a rhombus.



P Trapezoid properties

- Midsegment theorem: the segment joining the midpoints of the legs is parallel to the bases, and its length is $m = \frac{b_1 + b_2}{2}$.
- Isosceles trapezoid: the base angles are congruent, and the diagonals are congruent.
- Each leg makes two angles that are supplementary with its neighbor on the same leg: the angles at the ends of a leg add up to 180° .



P

Kite properties

- The diagonals are perpendicular.
- One diagonal (the axis of symmetry) bisects the other and bisects the angles at its ends.
- The two angles between the unequal sides are congruent.

!

Zyro says

On my planet we sort shapes by asking questions: Are there parallel sides? Are the sides equal? Are the angles right? Each answer moves you down the family tree!

6. Coordinate proofs with quadrilaterals

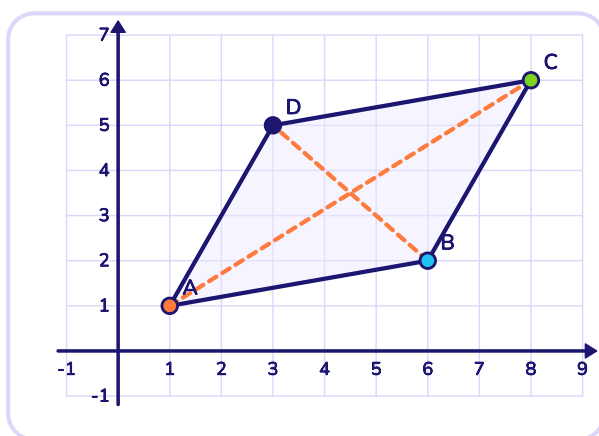
When a quadrilateral is given by the coordinates of its vertices, three tools do all the work:

- Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- Midpoint: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

- Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$. Parallel lines have equal slopes, and perpendicular lines have slopes whose product is -1 .

M Method: proving a type of quadrilateral

1. Plot the points to guess what the figure is.
2. Choose a criterion that matches your guess (for a parallelogram: the diagonals share a midpoint).
3. Compute exactly the distances, midpoints, or slopes you need.
4. State the conclusion with the theorem that justifies it.



E Example 4: a coordinate proof

Show that $A(1, 1)$, $B(6, 2)$, $C(8, 6)$, $D(3, 5)$ form a parallelogram. The midpoint of $\overline{AC}$ is $\left(\frac{1+8}{2}, \frac{1+6}{2}\right) = (4.5, 3.5)$. The midpoint of $\overline{BD}$ is $\left(\frac{6+3}{2}, \frac{2+5}{2}\right) = (4.5, 3.5)$. The diagonals bisect each other, so $ABCD$ is a parallelogram. It is not a rhombus, because $AB = \sqrt{26}$ but $BC = \sqrt{20}$.

Key takeaways

- Interior angles of an n -gon add up to $(n - 2) \cdot 180^\circ$; exterior angles always add up to 360° .
- A regular n -gon has exterior angles of $\frac{360^\circ}{n}$.
- Parallelogram: opposite sides and angles congruent, consecutive angles supplementary, diagonals bisect each other.
- Five criteria prove a parallelogram; one parallel pair or one congruent pair alone is not enough.
- Rectangle: congruent diagonals. Rhombus: perpendicular diagonals. Square: both.
- Trapezoid midsegment: $m = \frac{b_1 + b_2}{2}$. Kite: perpendicular diagonals, one bisected.
- In the coordinate plane, use distance, midpoint, and slope to prove a figure is what you claim.



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