

Quadratic Functions and Equations: practice solutions, Grade 11 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Reading the coefficients



(a) $a = -3$, $b = 12$, $c = -2$. Since $a < 0$, the parabola opens **downward**.

(b) The axis is $x = -\frac{12}{2 \cdot (-3)} = 2$. Then $f(2) = -12 + 24 - 2 = 10$, so the vertex is $(2, 10)$, a maximum.

(c) The y-intercept is $f(0) = c = -2$.

2 Reading the vertex form



Compare with $a(x - h)^2 + k$: $a = 2$, $h = -4$, $k = -9$. The vertex is $(-4, -9)$ and the axis is $x = -4$.

Since $a = 2 > 0$, the parabola opens upward and has a **minimum** value of -9 .

The y-intercept is $g(0) = 2 \cdot 16 - 9 = 23$.

3 Factoring simple trinomials



(a) Product **20**, sum **9**: the numbers are **4** and **5**, so $x^2 + 9x + 20 = (x + 4)(x + 5)$.

(b) Product **-12**, sum **-1**: the numbers are **-4** and **3**, so $(x - 4)(x + 3)$.

(c) Difference of squares: $(x - 8)(x + 8)$.

Solving $(x + 4)(x + 5) = 0$ gives $x = -4$ or $x = -5$.

4 Common factor



Factor out **5x**: $5x(x - 7) = 0$.

By the zero-product property, $5x = 0$ or $x - 7 = 0$, so $x = 0$ or $x = 7$.

Check: $5 \cdot 49 - 35 \cdot 7 = 245 - 245 = 0$.

5 Square root method



Divide by **3**: $(x - 2)^2 = 16$.

Take square roots: $x - 2 = 4$ or $x - 2 = -4$, so $x = 6$ or $x = -2$.

6 Using the discriminant



(a) $\Delta = 9 - 40 = -31 < 0$: no real solution.

(b) $\Delta = 36 - 36 = 0$: one real solution (a double root, $x = -3$).

(c) $\Delta = 49 + 32 = 81 > 0$: two real solutions, $x = \frac{-7 \pm 9}{4}$, that is $\frac{1}{2}$ and -4 .

7 Reading a graph for an inequality



- (a) At $x = -1$ and $x = 4$, the zeros of the two factors.
- (b) The graph is below the x-axis between the roots: $-1 < x < 4$.
- (c) The graph is on or above the axis outside the roots: $x \leq -1$ or $x \geq 4$.

8 Completing the square to get the vertex



Half of 8 is 4, and $4^2 = 16$. So $y = (x^2 + 8x + 16) - 16 + 3 = (x + 4)^2 - 13$.

The vertex is $(-4, -13)$.

9 Vertex form with a negative leading coefficient



Factor -2 from the first two terms: $y = -2(x^2 - 6x) - 7$.

Complete the square inside: $x^2 - 6x = (x - 3)^2 - 9$. So

$$y = -2((x - 3)^2 - 9) - 7 = -2(x - 3)^2 + 18 - 7 = -2(x - 3)^2 + 11.$$

The vertex is $(3, 11)$, and since $a < 0$ the maximum value is 11.

10 Factoring by grouping



We need two numbers with product $2 \cdot (-12) = -24$ and sum 5: 8 and -3 .

$$2x^2 + 8x - 3x - 12 = 2x(x + 4) - 3(x + 4) = (2x - 3)(x + 4).$$

So $x = \frac{3}{2}$ or $x = -4$.

11 Quadratic formula with a radical



$a = 1, b = -4, c = -3$, so $\Delta = 16 + 12 = 28$.

$$x = \frac{4 \pm \sqrt{28}}{2} = \frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7}.$$

Numerically $x \approx 4.65$ or $x \approx -0.65$.

12 A double root



One real solution means $\Delta = 0$: $k^2 - 4 \cdot 25 = 0$, so $k^2 = 100$ and $k = 10$ or $k = -10$.

If $k = 10$: $x^2 + 10x + 25 = (x + 5)^2 = 0$, so $x = -5$.

If $k = -10$: $(x - 5)^2 = 0$, so $x = 5$.

13 Building a function from its roots



Use $y = a(x - 2)(x - 7)$. At $x = 0$: $a \cdot (-2)(-7) = 14a = 28$, so $a = 2$.

$$y = 2(x^2 - 9x + 14) = 2x^2 - 18x + 28.$$

Check with $x = 3$: $2 \cdot 1 \cdot (-4) = -8$ and $18 - 54 + 28 = -8$. Both agree.

14 Two quadratic inequalities



(a) $x^2 - 5x + 4 = (x - 1)(x - 4)$. The parabola opens upward and is positive outside its roots: $x < 1$ or $x > 4$, that is $(-\infty, 1) \cup (4, \infty)$.

(b) $2x^2 + x - 6 = (2x - 3)(x + 2)$, with roots $\frac{3}{2}$ and -2 . The parabola is negative or zero between them: $-2 \leq x \leq \frac{3}{2}$, that is $\left[-2, \frac{3}{2}\right]$.

15 The patio



Let w be the width in feet. Then $w(w + 4) = 252$, so $w^2 + 4w - 252 = 0$.

$\Delta = 16 + 1008 = 1024 = 32^2$, so $w = \frac{-4 \pm 32}{2}$, that is $w = 14$ or $w = -18$. A width cannot be negative, so $w = 14$.

The patio is **14** ft wide and **18** ft long (about **4.3** m by **5.5** m). Check: $14 \cdot 18 = 252$.

16 Complex solutions



$\Delta = 36 - 52 = -16$, so $\sqrt{\Delta} = 4i$. Then $x = \frac{6 \pm 4i}{2} = 3 \pm 2i$.

Check $3 + 2i$: $(3 + 2i)^2 = 9 + 12i - 4 = 5 + 12i$; $-6(3 + 2i) = -18 - 12i$; the sum $5 + 12i - 18 - 12i + 13 = 0$.

17 A rock thrown from a cliff



(a) Solve $-16t^2 + 24t + 40 = 0$. Dividing by -8 : $2t^2 - 3t - 5 = 0$, which factors as $(2t - 5)(t + 1) = 0$. So $t = 2.5$ or $t = -1$. Time cannot be negative: the rock hits the ground after **2.5 seconds**.

(b) The vertex is at $t = -\frac{24}{2 \cdot (-16)} = 0.75$ s, where $h(0.75) = -9 + 18 + 40 = 49$. The maximum height is **49** feet (about **14.9** m), reached after **0.75** s.

18 Revenue and price



(a) $R(p) = -4p^2 + 120p$. The vertex is at $p = -\frac{120}{2 \cdot (-4)} = 15$, and

$R(15) = 15 \cdot 60 = 900$. The best price is **\$15**, for a revenue of **\$900**.

(b) Solve $-4p^2 + 120p \geq 800$, so $-4p^2 + 120p - 800 \geq 0$. Divide by -4 and reverse the inequality: $p^2 - 30p + 200 \leq 0$, that is $(p - 10)(p - 20) \leq 0$. So $10 \leq p \leq 20$.

The revenue is at least \$800 for prices between \$10 and \$20.

19 A line and a parabola



Set the expressions equal: $x^2 - 2x - 3 = x + 1$, so $x^2 - 3x - 4 = 0$, that is $(x - 4)(x + 1) = 0$.

$x = 4$ gives $y = 5$; $x = -1$ gives $y = 0$.

The intersection points are $(4, 5)$ and $(-1, 0)$. Check on the parabola: $16 - 8 - 3 = 5$ and $1 + 2 - 3 = 0$.

20 Roots that differ by 6



Call the roots r and $r + 6$. Their sum is 10 (the opposite of the coefficient of x), so $2r + 6 = 10$ and $r = 2$.

The roots are 2 and 8 , and $c = 2 \cdot 8 = 16$.

Check with the discriminant: $100 - 4 \cdot 16 = 36$, and $\frac{10 \pm 6}{2}$ gives 8 and 2 .

21 Always positive



The parabola opens upward, so the expression is always positive exactly when the graph stays above the x-axis, that is, when there is no real root: $\Delta < 0$.

$\Delta = 16 - 4k < 0$ gives $k > 4$.

With the vertex: $x^2 - 4x + k = (x - 2)^2 + k - 4$. The minimum value is $k - 4$, which is positive when $k > 4$. Both methods agree.



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