

Quadratic Functions and Graphs: math lesson, Grade 9 – download the PDF

Throw a ball, launch a rocket, or aim a basketball shot: the path is a smooth curve called a parabola. Quadratic functions describe these curves exactly. In this chapter you will learn to read them, graph them, move them around, and use them to find the highest point, the lowest point, and the moments when something hits the ground.

1. What is a quadratic function?

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Standard form

A **quadratic function** is a function that can be written as

$$f(x) = ax^2 + bx + c,$$

where a , b , and c are real numbers and $a \neq 0$.

The number a is the **leading coefficient**, b is the coefficient of the linear term, and c is the **constant term**. The highest power of x is 2, which is why we say “quadratic.” If $a = 0$ the x^2 term vanishes and you are left with a line, so a can never be 0.

For example, $f(x) = 3x^2 - 5x + 2$ has $a = 3$, $b = -5$, and $c = 2$. The function $g(x) = -x^2 + 4$ has $a = -1$, $b = 0$, and $c = 4$ because the missing x -term has a coefficient of 0.



Watch the signs

Always read the sign in front of each term. In $y = 6 - 2x - x^2$, first rewrite it in standard form, $y = -x^2 - 2x + 6$. Then $a = -1$, $b = -2$, $c = 6$.

2. Parabolas, vertex, and axis of symmetry

The graph of a quadratic function is a **parabola**. Every parabola has a turning point, the **vertex**, and a vertical line through it, the **axis of symmetry**, that splits the curve into two mirror-image halves.



Key facts for $f(x) = ax^2 + bx + c$

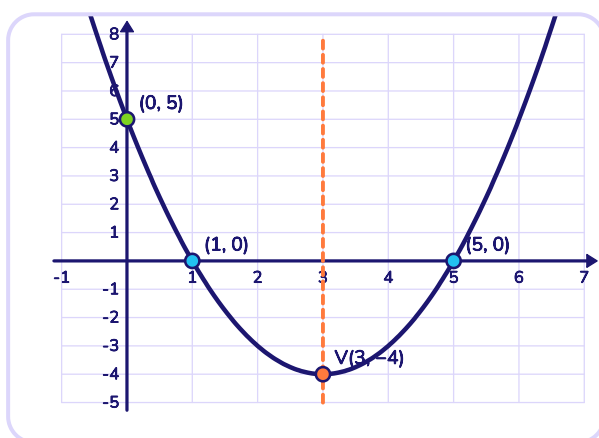
- If $a > 0$, the parabola opens **upward** and the vertex is the lowest point (a **minimum**).
- If $a < 0$, the parabola opens **downward** and the vertex is the highest point (a **maximum**).
- The axis of symmetry is the line $x = -\frac{b}{2a}$.
- The vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
- The y -intercept is the point $(0, c)$.

E**Example 1: finding the vertex**

Let $f(x) = x^2 - 6x + 5$. Here $a = 1$, $b = -6$, $c = 5$.

Axis of symmetry: $x = -\frac{-6}{2 \cdot 1} = 3$.

Vertex: $f(3) = 9 - 18 + 5 = -4$, so the vertex is $(3, -4)$. Since $a > 0$, it is a minimum. The y -intercept is $(0, 5)$.



In the graph above the dashed line is the axis $x = 3$. Notice that $(0, 5)$ has a mirror partner $(6, 5)$ on the other side of the axis.

3. Graphing a quadratic function

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Method: graph a parabola in five steps

1. Look at the sign of a to decide whether the parabola opens up or down.
2. Find the axis of symmetry $x = -\frac{b}{2a}$.
3. Substitute that x -value to find the vertex.
4. Plot the y -intercept $(0, c)$ and its mirror image across the axis.
5. Add one or two more points (for example the x -intercepts or a table value), then draw a smooth curve through them. Never connect the points with straight segments.

A table of values is a good way to check your work. For $y = x^2 - 6x + 5$:

x	0	1	2	3	4	5	6
y	5	0	-3	-4	-3	0	5

The values rise and fall symmetrically around $x = 3$, which confirms the axis of symmetry.

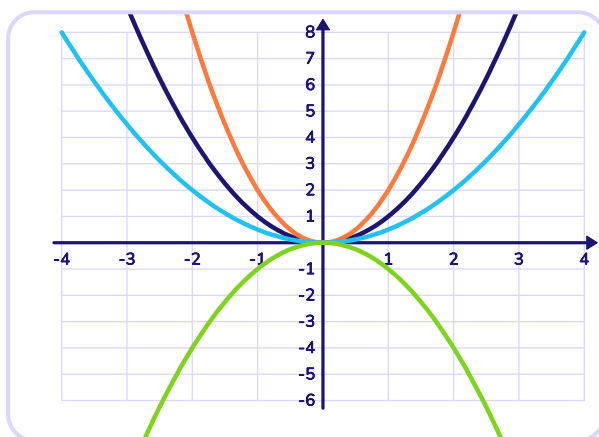
4. Transformations of parabolas

The simplest parabola is the parent function $y = x^2$, with its vertex at the origin. Every other parabola can be obtained from it by shifting, stretching, shrinking, or flipping.



Transformations of $y = x^2$

- $y = x^2 + k$: shifts the graph **up** k units (down if $k < 0$).
- $y = (x - h)^2$: shifts the graph **right** h units (left if $h < 0$).
- $y = ax^2$ with $|a| > 1$: **stretches** the graph vertically, so the parabola is narrower.
- $y = ax^2$ with $0 < |a| < 1$: **shrinks** the graph vertically, so the parabola is wider.
- $y = -x^2$: **reflects** the graph across the x -axis, so it opens downward.



In this figure the dark blue curve is $y = x^2$, the orange curve $y = 2x^2$ is narrower, the cyan curve $y = 0.5x^2$ is wider, and the green curve $y = -x^2$ opens downward.



Common mistake

The sign inside the parentheses works backward from what you might expect.

$y = (x - 2)^2$ moves the graph **right** 2, and $y = (x + 3)^2$ moves it **left** 3, because the squared part is 0 when $x = 2$ or $x = -3$.

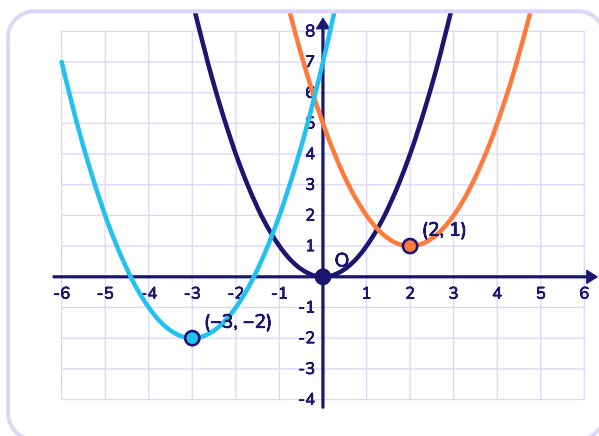
5. Vertex form

D Vertex form

A quadratic function written as

$$f(x) = a(x - h)^2 + k$$

is in **vertex form**. Its vertex is (h, k) , its axis of symmetry is $x = h$, and a gives the direction and width.



The orange parabola is $y = (x - 2)^2 + 1$: the parent graph moved right 2 and up 1. The cyan parabola is $y = (x + 3)^2 - 2$: moved left 3 and down 2.

E Example 2: from standard form to vertex form

Rewrite $f(x) = x^2 - 6x + 5$ by completing the square. Half of -6 is -3 , and $(-3)^2 = 9$:

$$f(x) = (x^2 - 6x + 9) - 9 + 5 = (x - 3)^2 - 4.$$

The vertex is $(3, -4)$, exactly what we found in Example 1.

E**Example 3: writing an equation from a vertex and a point**

A parabola has vertex $(1, 8)$ and passes through $(3, 0)$. Start with $y = a(x - 1)^2 + 8$ and substitute the point: $0 = a(2)^2 + 8$, so $4a = -8$ and $a = -2$. The function is $y = -2(x - 1)^2 + 8$.

!**Zyro's tip**

On my planet we say: “read the vertex straight from the parentheses, and flip the sign of the number next to x .” In $y = 3(x + 7)^2 - 4$, the vertex is $(-7, -4)$.

6. Roots, zeros, and x-intercepts

A **zero** (or **root**) of f is an x -value for which $f(x) = 0$. On the graph, zeros are the x -intercepts, where the parabola crosses the x -axis. A parabola can have two, one, or no x -intercepts.

M**Method: solve $ax^2 + bx + c = 0$**

1. **Factor** when you can, then use the zero product property: if $pq = 0$, then $p = 0$ or $q = 0$.
2. For a vertex form, isolate the square and take square roots (do not forget the $\pm$).
3. Otherwise use the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The expression $b^2 - 4ac$ is the **discriminant**. If it is positive there are two real zeros, if it is 0 there is exactly one, and if it is negative the parabola never touches the x -axis.

E Example 4: zeros of a function in vertex form

Find the zeros of $g(x) = -2(x - 1)^2 + 8$. Set it equal to 0: $-2(x - 1)^2 = -8$, so $(x - 1)^2 = 4$ and $x - 1 = \pm 2$. The zeros are $x = 3$ and $x = -1$.

E Example 5: zeros by factoring

Solve $x^2 - 6x + 5 = 0$. Factor: $(x - 1)(x - 5) = 0$, so $x = 1$ or $x = 5$. These match the x -intercepts in the first graph.

7. Maximum and minimum values

The y -coordinate of the vertex is the **minimum value** when the parabola opens up and the **maximum value** when it opens down. The value is the output, not the x -coordinate. For $f(x) = x^2 - 6x + 5$, the minimum *value* is -4 , and it occurs at $x = 3$. The **range** of this function is $y \geq -4$.

For $g(x) = -2(x - 1)^2 + 8$, the maximum value is 8, reached at $x = 1$, and the range is $y \leq 8$. The domain of every quadratic function is all real numbers.

8. Quadratic models

Quadratic functions model situations where a quantity rises and then falls, or where a product of two linked quantities is involved, such as area and revenue. The vertex tells you the best possible result.

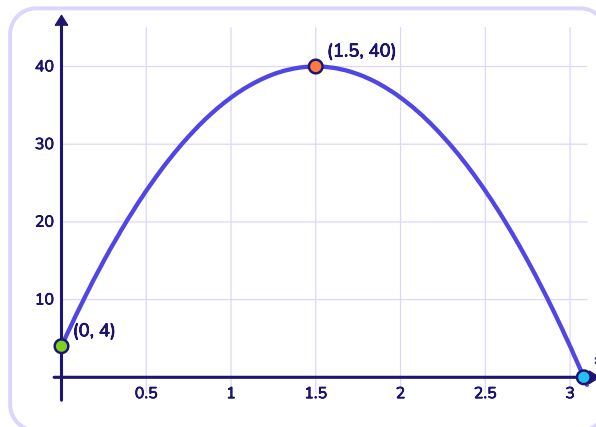
E**Example 6: a ball in the air**

A ball is thrown upward from a height of 4 feet. Its height in feet after t seconds is $h(t) = -16t^2 + 48t + 4$.

Highest point: $t = -\frac{48}{2(-16)} = 1.5$ seconds, and $h(1.5) = -36 + 72 + 4 = 40$ feet (about 12.2 meters).

Landing time: solve $-16t^2 + 48t + 4 = 0$. The discriminant is

$48^2 - 4(-16)(4) = 2560$, so $t = \frac{-48 - \sqrt{2560}}{-32} \approx 3.08$ seconds (the other root is negative, so it is rejected).

**Check that your answer makes sense**

A time can never be negative, and a length can never be negative. When a quadratic gives two solutions, ask which one fits the real situation.

Key takeaways

- A quadratic function has the standard form $f(x) = ax^2 + bx + c$ with $a \neq 0$, and its graph is a parabola.
- The axis of symmetry is $x = -\frac{b}{2a}$; substitute it into f to get the vertex.
- If $a > 0$ the parabola opens up (minimum); if $a < 0$ it opens down (maximum).
- Vertex form $f(x) = a(x - h)^2 + k$ shows the vertex (h, k) and the transformations of $y = x^2$ at a glance.
- Zeros are the x -intercepts: factor, take square roots, or use the quadratic formula; the discriminant counts the real zeros.
- The maximum or minimum *value* is the y -coordinate of the vertex.
- In a model, always reject answers that do not fit the context, and give units.



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