

Quadratic Functions and Equations: math lesson, Grade 11 – download the PDF

A basketball arcing toward the hoop, the cable of a suspension bridge, a satellite dish, the profit of a small business: all of them follow a **parabola**. In this chapter you will master the algebra behind it. You will move between the standard form and the vertex form of a quadratic function, solve quadratic equations in four different ways, use the discriminant to predict the number of solutions before you start, solve quadratic inequalities, and model real situations.

1. Standard form and the shape of a parabola

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Quadratic function

A quadratic function is a function that can be written in **standard form**

$$f(x) = ax^2 + bx + c$$

where a , b and c are real numbers and $a \neq 0$. Its graph is a curve called a **parabola**.

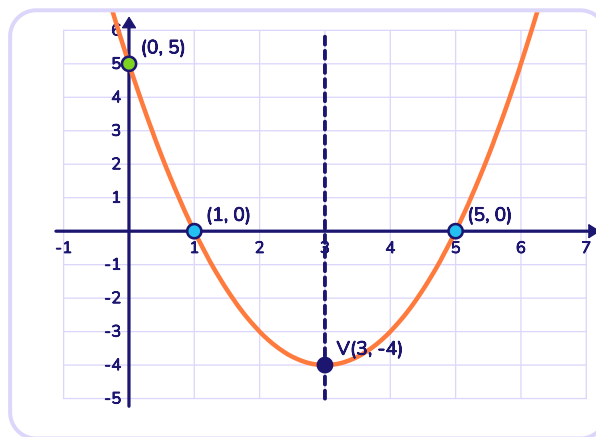
The three coefficients each tell you something about the graph:

- If $a > 0$ the parabola opens upward and its vertex is the **minimum** point. If $a < 0$ it opens downward and the vertex is the **maximum** point.
- The number c is the **y-intercept**, because $f(0) = c$.

- The vertical line $x = -\frac{b}{2a}$ is the **axis of symmetry**. The vertex lies on this line, so its y-coordinate is $f\left(-\frac{b}{2a}\right)$.

E Example 1: reading a parabola

Take $f(x) = x^2 - 6x + 5$. Here $a = 1 > 0$, so the parabola opens upward. The axis of symmetry is $x = -\frac{-6}{2 \cdot 1} = 3$ and $f(3) = 9 - 18 + 5 = -4$, so the vertex is $(3, -4)$. The y-intercept is 5.



2. Vertex form

D Vertex form

Every quadratic function can be written as

$$f(x) = a(x - h)^2 + k$$

The vertex is the point (h, k) and the axis of symmetry is $x = h$. The coefficient a is the same as in standard form.

Vertex form is the best form for graphing and for finding a maximum or minimum, because the vertex can be read directly. For $f(x) = x^2 - 6x + 5$ from the example above we can write

$f(x) = (x - 3)^2 - 4$: the vertex is $(3, -4)$. Notice the sign: $(x - 3)$ gives $h = +3$, while $(x + 2)$ would give $h = -2$.

The number $|a|$ controls how narrow the parabola is: the larger $|a|$, the steeper and narrower the curve. A negative a flips it upside down.

3. Factoring and the zero-product property

Factoring is the fastest way to solve a quadratic equation when it works. It relies on one idea.

P

Zero-product property

If $A \cdot B = 0$, then $A = 0$ or $B = 0$. So to solve $ax^2 + bx + c = 0$, write the left side as a product of factors, set each factor equal to zero, and solve.

M

Method: factoring a trinomial

1. Move every term to one side so that the equation reads $ax^2 + bx + c = 0$.
2. Take out a common factor first if there is one.
3. If $a = 1$, find two numbers whose product is c and whose sum is b .
4. If $a \neq 1$, find two numbers whose product is ac and whose sum is b , split the middle term, and factor by grouping.
5. Look for special patterns: $A^2 - B^2 = (A - B)(A + B)$.
6. Set each factor equal to zero and solve.

E**Example 2: factoring with $a \neq 1$**

Solve $6x^2 + 7x - 3 = 0$. We need two numbers with product $6 \cdot (-3) = -18$ and sum 7: they are 9 and -2 . Then

$$6x^2 + 9x - 2x - 3 = 3x(2x + 3) - (2x + 3) = (3x - 1)(2x + 3).$$

So $3x - 1 = 0$ or $2x + 3 = 0$, which gives $x = \frac{1}{3}$ or $x = -\frac{3}{2}$.

**Common mistake**

You may only use the zero-product property when the other side is **exactly zero**. From $(x - 2)(x + 5) = 8$ you cannot conclude that $x - 2 = 8$!

4. Completing the square

Not every quadratic factors nicely. Completing the square always works, and it is also how you convert standard form into vertex form. The key fact is

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2.$$

M**Method: solving by completing the square**

1. Divide the equation by a so that the coefficient of x^2 is 1.
2. Move the constant term to the right side.
3. Add $\left(\frac{b}{2}\right)^2$ to **both** sides.
4. Write the left side as a perfect square.
5. Take the square root of both sides, remembering the $\pm$ sign, and solve for x .

E**Example 3: completing the square**

Solve $2x^2 + 12x - 14 = 0$. Divide by 2: $x^2 + 6x - 7 = 0$. Move the constant:

$x^2 + 6x = 7$. Add $\left(\frac{6}{2}\right)^2 = 9$ to both sides: $x^2 + 6x + 9 = 16$, so $(x + 3)^2 = 16$.

Then $x + 3 = \pm 4$, which gives $x = 1$ or $x = -7$.

To convert a function, use the same idea. For $g(x) = x^2 + 8x + 3$, write
 $g(x) = (x^2 + 8x + 16) - 16 + 3 = (x + 4)^2 - 13$. The vertex is $(-4, -13)$.

5. The quadratic formula

Completing the square on the general equation $ax^2 + bx + c = 0$ gives a formula that solves **every** quadratic equation.

P**Quadratic formula**

If $ax^2 + bx + c = 0$ with $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

E**Example 4: using the formula**

Solve $3x^2 - 5x - 1 = 0$. Here $a = 3$, $b = -5$, $c = -1$, so

$$x = \frac{5 \pm \sqrt{25 + 12}}{6} = \frac{5 \pm \sqrt{37}}{6}.$$

The exact solutions are $\frac{5 + \sqrt{37}}{6} \approx 1.85$ and $\frac{5 - \sqrt{37}}{6} \approx -0.18$.

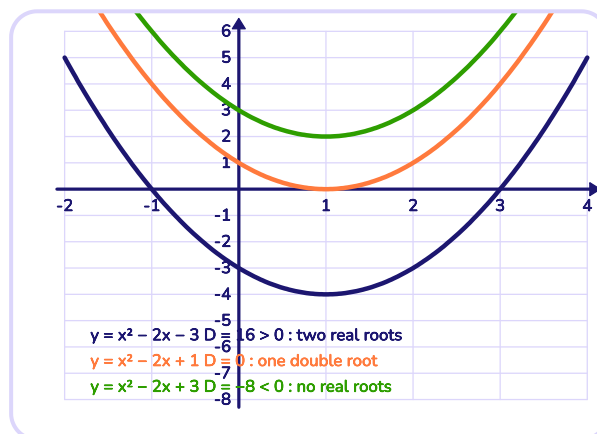
!**Zyro's tip**

Which method should you pick? Look for a square root first ($x^2 = k$), try factoring if the numbers are small, and keep the quadratic formula as your reliable tool for everything else. Put parentheses around negative values of b before you square them!

6. The discriminant**D****Discriminant**

The expression $\Delta = b^2 - 4ac$ under the square root is called the discriminant. Its sign tells you what kind of solutions the equation has, without solving it.

Value of Δ	Solutions of $ax^2 + bx + c = 0$	Graph
$\Delta > 0$	two distinct real solutions	crosses the x-axis twice
$\Delta = 0$	one real solution (a double root)	touches the x-axis at the vertex
$\Delta < 0$	no real solution, two complex solutions	never meets the x-axis



When $\Delta < 0$, the solutions are complex numbers that involve $i = \sqrt{-1}$. For $x^2 - 2x + 3 = 0$ we get $\Delta = 4 - 12 = -8$, so $x = \frac{2 \pm \sqrt{-8}}{2} = 1 \pm i\sqrt{2}$. The two solutions are always complex conjugates of each other.

7. Quadratic inequalities

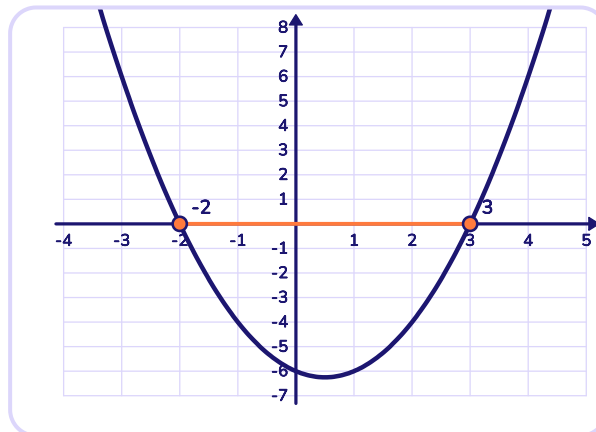
To solve an inequality such as $x^2 - x - 6 < 0$, you look at where the parabola lies below, above or on the x-axis.

M**Method**

1. Move everything to one side so that the other side is **0**.
2. Find the roots of the related equation.
3. Sketch the parabola or make a sign chart. A parabola that opens upward is negative between its roots and positive outside them.
4. Write the solution as an interval or union of intervals. Use brackets for $\leq$ and $\geq$, parentheses for $<$ and $>$.

E**Example 5: a quadratic inequality**

Solve $x^2 - x - 6 < 0$. Since $x^2 - x - 6 = (x - 3)(x + 2)$, the roots are -2 and 3 . The parabola opens upward, so it lies below the x-axis between the roots. The solution is $-2 < x < 3$, that is $(-2, 3)$.

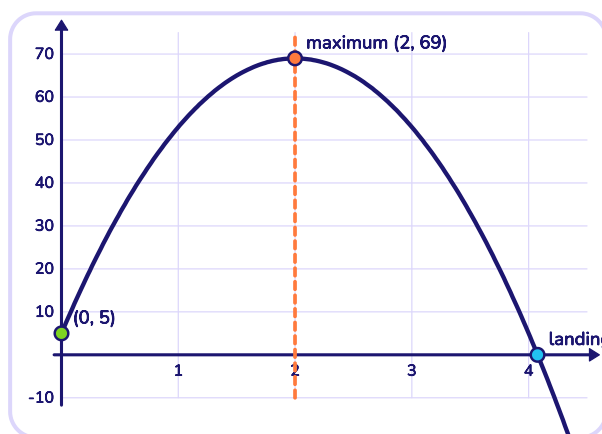
**8. Modeling with quadratic functions**

Quadratics model any situation where one quantity depends on the **square** of another: the area of a rectangle, the path of a thrown object, or the revenue of a store when the price changes. Two questions come back again and again: *where is the maximum or minimum* (the vertex) and *when does the quantity reach a given value* (a quadratic equation).

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Example 6: a ball in the air

A ball is thrown upward from a height of **5** feet (about **1.5** m). Its height in feet after t seconds is $h(t) = -16t^2 + 64t + 5$. The vertex is at $t = -\frac{64}{2 \cdot (-16)} = 2$ seconds, where $h(2) = -64 + 128 + 5 = 69$ feet (about **21** m). The ball lands when $h(t) = 0$: by the quadratic formula $t = \frac{64 \pm \sqrt{4416}}{32}$, and the positive solution is $t \approx 4.08$ seconds. The ball is at least **53** feet high when $-16t^2 + 64t - 48 \geq 0$, that is $t^2 - 4t + 3 \leq 0$, so $1 \leq t \leq 3$.



When you answer a word problem, always check that your solution makes sense: a negative time or length must be rejected, and a final answer needs a sentence with units.

Key takeaways

- Standard form $ax^2 + bx + c$: the axis of symmetry is $x = -\frac{b}{2a}$ and the y-intercept is c .
- Vertex form $a(x - h)^2 + k$: the vertex is (h, k) . Completing the square converts standard form into vertex form.
- Factoring plus the zero-product property solves an equation that equals 0.
- The quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ always works.
- $\Delta > 0$: two real solutions; $\Delta = 0$: one; $\Delta < 0$: two complex solutions.
- A quadratic inequality is solved with the roots and the direction of the parabola.
- In a model, the vertex gives the best value, and you must reject solutions that do not fit the situation.



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