

The Pythagorean Theorem: math lesson, Grade 8 – download the PDF

Builders use a tiny trick to check that a corner is perfectly square, and surveyors use the same idea to measure across a lake without getting wet. The trick is the Pythagorean theorem, one of the most useful facts in all of geometry. In this chapter you will learn what it says, why it is true, how to use it to find missing lengths, how to test for a right angle, and how it reaches into the coordinate plane and into three dimensions.

1. The statement of the theorem

D Right triangle, legs, hypotenuse

A **right triangle** has one angle of exactly 90° . The side opposite that angle is the longest side and is called the **hypotenuse**. The two sides that form the right angle are the **legs**.

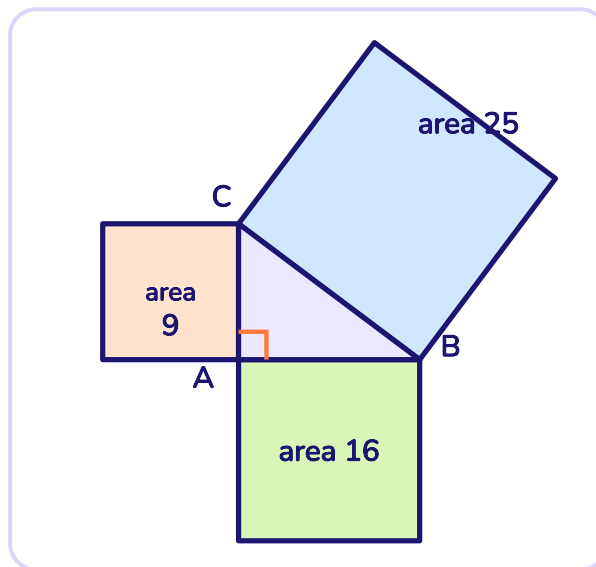


Pythagorean theorem

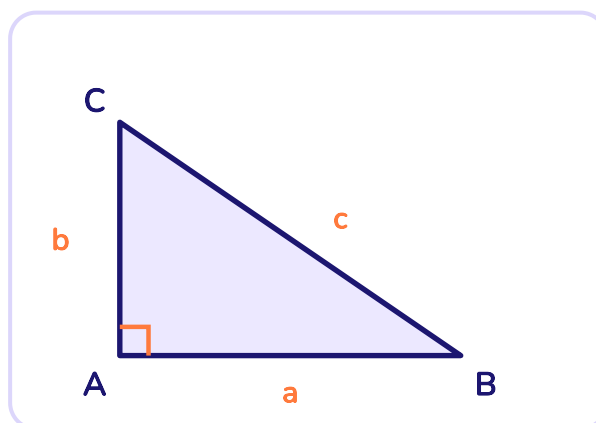
In a right triangle with legs a and b and hypotenuse c :

$$a^2 + b^2 = c^2$$

In words: the area of the square on the hypotenuse equals the sum of the areas of the squares on the two legs.



The picture shows a triangle with legs 3 and 4. The squares on the legs have areas $3^2 = 9$ and $4^2 = 16$. Together they make 25, which is exactly the area of the square on the hypotenuse, so the hypotenuse is 5.



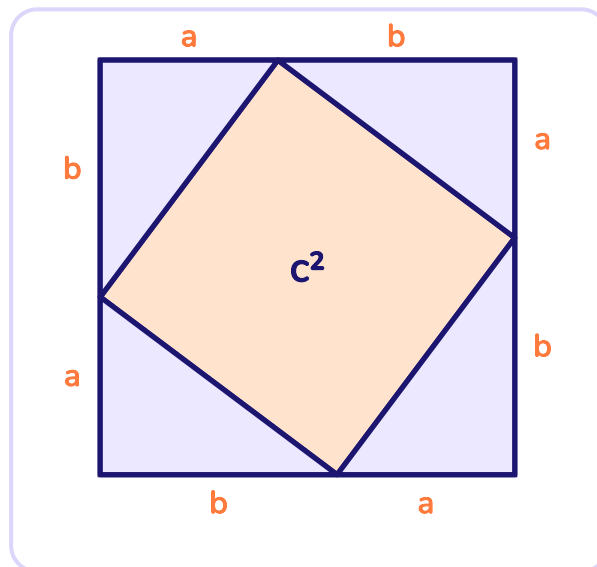


Watch out

The letter ***c*** always names the hypotenuse, the side facing the right angle. It is not always the side drawn at the bottom or on the right. Find the right angle first, then decide which side is opposite it.

2. Why the theorem is true

You do not have to take the theorem on faith. Here is a short proof using areas. Take four identical right triangles with legs ***a*** and ***b*** and hypotenuse ***c***. Place them inside a large square of side ***a + b***, as in the figure. Their hypotenuses form a tilted square in the middle with side ***c***.



Now compute the area of the large square in two different ways.

- As one big square: $(a + b)^2 = a^2 + 2ab + b^2$.
- As four triangles plus the middle square: $4 \cdot \frac{1}{2}ab + c^2 = 2ab + c^2$.

The two areas are equal, so $a^2 + 2ab + b^2 = 2ab + c^2$. Subtract $2ab$ from both sides and you get $a^2 + b^2 = c^2$. That is the theorem.

3. Finding the hypotenuse

M Method: find the hypotenuse

1. Identify the right angle and the side opposite it. Call that side c .
2. Write $a^2 + b^2 = c^2$ and substitute the two legs.
3. Add the squares to get c^2 .
4. Take the positive square root: $c = \sqrt{c^2}$.
5. Write the answer with its unit, and check that it is longer than each leg.

E Example 1: a perfect square

A right triangle has legs 9 cm and 12 cm. Then $c^2 = 9^2 + 12^2 = 81 + 144 = 225$, so $c = \sqrt{225} = 15$ cm.

E Example 2: not a perfect square

The legs are 5 in and 7 in. Then $c^2 = 25 + 49 = 74$, so $c = \sqrt{74} \approx 8.60$ in. The exact value is $\sqrt{74}$; the decimal is a rounded estimate.

4. Finding a missing leg

When the hypotenuse is known, rearrange the equation: $a^2 = c^2 - b^2$. You *subtract* the squares, because the hypotenuse is the biggest side.

E Example 3: a missing leg

The hypotenuse is 17 m and one leg is 8 m. Then $a^2 = 17^2 - 8^2 = 289 - 64 = 225$, so $a = 15$ m.

⚠ Watch out

Never add the squares when you are looking for a leg, and never forget the last step of taking the square root. A common slip is to answer 225 instead of 15.

! Zyro's tip

On my planet we memorize a few whole-number triples so that we can spot them instantly. Try these: 3-4-5, 5-12-13, 8-15-17, 7-24-25 and 20-21-29. Any multiple of a triple works too, so 6-8-10 and 9-12-15 are right triangles as well.

Legs a, b	$a^2 + b^2$	Hypotenuse c
3 and 4	$9 + 16 = 25$	5
5 and 12	$25 + 144 = 169$	13
8 and 15	$64 + 225 = 289$	17
7 and 24	$49 + 576 = 625$	25
20 and 21	$400 + 441 = 841$	29

5. The converse: testing for a right angle

The theorem also works backward. If you know three side lengths, you can find out whether the triangle has a right angle.

P Converse of the Pythagorean theorem

If the sides of a triangle satisfy $a^2 + b^2 = c^2$, where c is the longest side, then the triangle is a right triangle, and the right angle is opposite the side c .

If $a^2 + b^2 \neq c^2$, the triangle is not a right triangle. Always test the *longest* side as c .

E Example 4: a right triangle

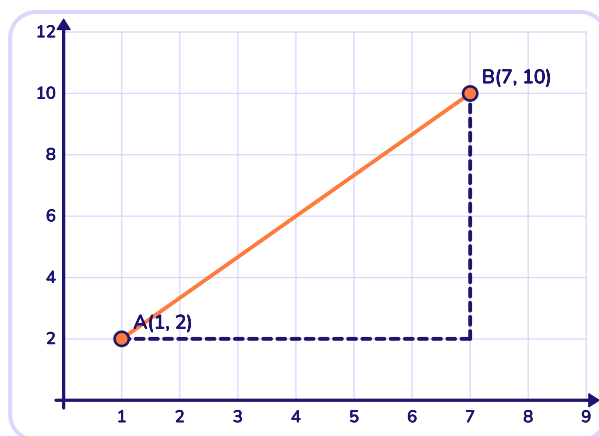
Sides 20, 21 and 29. Compare $20^2 + 21^2 = 400 + 441 = 841$ with $29^2 = 841$. They are equal, so the triangle is right.

E Example 5: not a right triangle

Sides 6, 8 and 11. We get $6^2 + 8^2 = 100$, but $11^2 = 121$. Since $100 \neq 121$, the triangle has no right angle.

6. Distance between two points

In the coordinate plane, a segment that is neither horizontal nor vertical is the hypotenuse of a right triangle whose legs run parallel to the axes. The legs have lengths $|x_2 - x_1|$ and $|y_2 - y_1|$.



P Distance formula

The distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

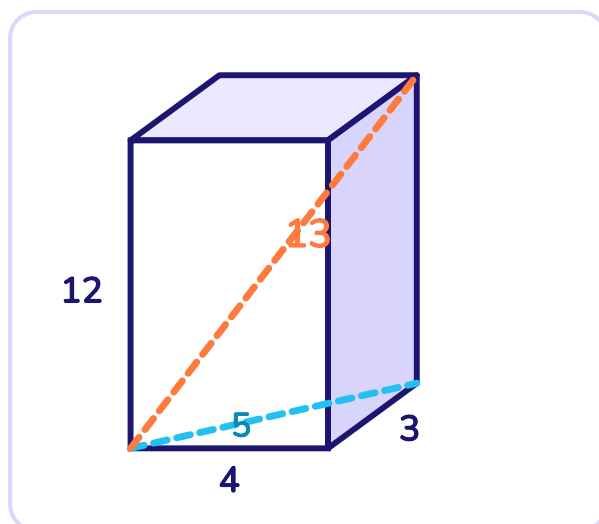
E Example 6: distance in the plane

For $A(1, 2)$ and $B(7, 10)$ the legs are $7 - 1 = 6$ and $10 - 2 = 8$. So

$AB = \sqrt{36 + 64} = \sqrt{100} = 10$ units. For $P(-3, 2)$ and $Q(4, -1)$, the legs are 7 and 3, so $PQ = \sqrt{49 + 9} = \sqrt{58} \approx 7.62$ units.

I 7. The theorem in three dimensions

A rectangular prism with length ℓ , width w and height h has a **space diagonal** that goes from one corner through the inside to the opposite corner. Use the theorem twice: first on the base, then on a vertical triangle.



P Space diagonal

$$d = \sqrt{\ell^2 + w^2 + h^2}$$

E Example 7: diagonal of a box

Take a box 4 in by 3 in by 12 in. The base diagonal is $\sqrt{4^2 + 3^2} = 5$ in. The space diagonal is then $\sqrt{5^2 + 12^2} = 13$ in. Check with the formula:

$$\sqrt{16 + 9 + 144} = \sqrt{169} = 13.$$

8. Real-world applications

Whenever a situation hides a right angle, such as a wall and the ground, the sides of a field, or the corner of a screen, the theorem links the three lengths. Draw a sketch, mark the right angle, label the known lengths, and then choose between adding squares (hypotenuse) or subtracting squares (leg).

E**Example 8: a baseball diamond**

The bases of a baseball diamond form a square 90 ft on each side. The throw from home plate to second base is the diagonal of that square:

$$d = \sqrt{90^2 + 90^2} = \sqrt{16,200} \approx 127.3 \text{ ft, which is about 38.8 m.}$$

M**Method: solving a word problem**

1. Sketch the situation and mark the right angle.
2. Label the known lengths with units.
3. Decide which side is the hypotenuse.
4. Write the equation, solve it, and answer in a full sentence.

Key takeaways

- In a right triangle, $a^2 + b^2 = c^2$, where c is the hypotenuse, the side opposite the right angle.
- To find the hypotenuse, add the squares of the legs and take the square root.
- To find a leg, subtract the square of the known leg from the square of the hypotenuse, then take the square root.
- Converse: if $a^2 + b^2 = c^2$ for the longest side c , the triangle is right.
- Distance formula: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- Space diagonal of a box: $d = \sqrt{\ell^2 + w^2 + h^2}$.
- Always sketch, label, and include units in your final answer.



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