

# Proportional Relationships and Slope: math lesson, Grade 8 – download the PDF

A skateboard ramp, a phone charging, a runner on a track: in each case one quantity changes at a steady pace compared with another. In this chapter you will learn to measure that pace with a single number, the **unit rate**, and to read it on a graph as the **slope** of a line.

## 1. Proportional relationships and unit rate

Two quantities are in a **proportional relationship** when one is always a fixed multiple of the other. If  $x$  and  $y$  are proportional, the quotient  $\frac{y}{x}$  is the same for every pair of values (with  $x \neq 0$ ).

### D Definition

The **unit rate** is the amount of  $y$  for exactly **1 unit** of  $x$ . In a proportional relationship it is called the **constant of proportionality**  $k$ , and the relationship is written  $y = kx$ .

### E Example 1: finding a unit rate

A hiker walks 7.5 miles in 3 hours. The unit rate is  $\frac{7.5}{3} = 2.5$  miles per hour, so the distance  $d$  after  $t$  hours is  $d = 2.5t$ . After 4 hours she has walked  $2.5 \times 4 = 10$  miles.



### Watch the order

The unit rate depends on which quantity goes on top. Miles per hour is  $\frac{\text{miles}}{\text{hours}}$ ; hours per mile is the reciprocal. Always read the question: “per” tells you what goes in the denominator.

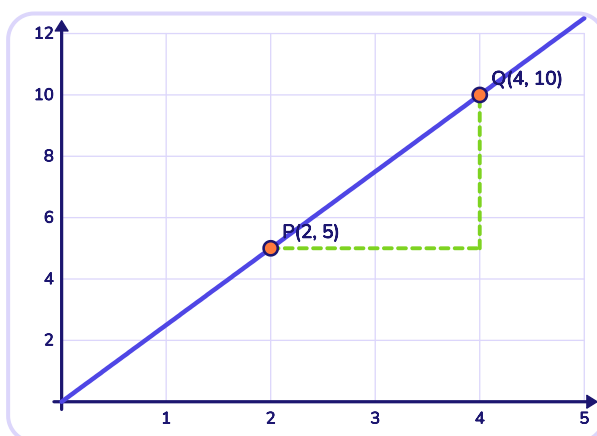
## 2. Graphing proportional relationships

To graph a proportional relationship, make a table of values, plot the ordered pairs on the coordinate plane, and connect them. The result always has two features.



### Property

The graph of a proportional relationship is a **straight line that passes through the origin  $(0, 0)$** . If the line is straight but misses the origin, or if it is curved, the relationship is not proportional.



On the graph above, the points  $P(2, 5)$  and  $Q(4, 10)$  both satisfy  $d = 2.5t$ . Notice that when the time doubles from 2 to 4 hours, the distance doubles too, from 5 to 10 miles.



### Common mistake

The line  $y = 2x + 1$  is straight and increasing, but it crosses the  $y$ -axis at 1, not at 0. When  $x = 1$ ,  $y = 3$  and the ratio is 3; when  $x = 2$ ,  $y = 5$  and the ratio is 2.5. The ratios differ, so this is **not** proportional.

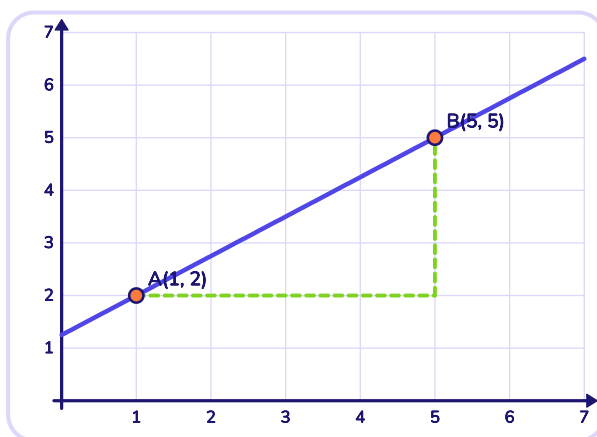
## 3. Slope as rate of change

Between two points on a line, we compare how far we go up or down (the **rise**) with how far we go across (the **run**).



### Definition

The **slope** of a line is  $m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$ . It is the **rate of change** of  $y$  with respect to  $x$ .



On the line above, going from  $A(1, 2)$  to  $B(5, 5)$  means moving 4 units to the right (run) and 3 units up (rise), so the slope is  $\frac{3}{4}$ . For a proportional relationship  $y = kx$ , the slope equals the unit rate:  $m = k$ . In the hiker example, the slope 2.5 means “2.5 miles for each hour.”

A line that goes up from left to right has a **positive** slope; one that goes down has a **negative** slope.

## 4. Slope from a graph

M

### Method: reading a slope on a graph

1. Choose two points that sit exactly on grid intersections.
2. Starting from the left point, count the **run** to the right.
3. Count the **rise**: up is positive, down is negative.
4. Write  $m = \frac{\text{rise}}{\text{run}}$  and simplify. Give the unit if the situation has one.

E

### Example 2: slope from a graph

In the hiker graph, from  $P(2, 5)$  to  $Q(4, 10)$  the run is 2 and the rise is 5. The slope is  $\frac{5}{2} = 2.5$  miles per hour, matching the unit rate.

!

### Zyro's tip

On my home planet we say: "run first, then rise." Walk across the grid before you climb, and you will never mix up the numerator and the denominator!

## 5. Slope from two points

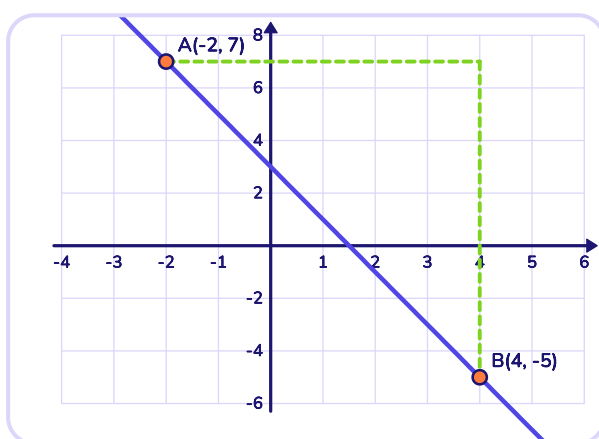
When you only have coordinates, there is no need to draw anything.

## P Slope formula

For points  $(x_1, y_1)$  and  $(x_2, y_2)$  with  $x_1 \neq x_2$ :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Subtract the coordinates in the **same order** on top and bottom. Swapping the two points changes both signs, so the quotient stays the same.



## E Example 3: a negative slope

For  $A(-2, 7)$  and  $B(4, -5)$ :  $m = \frac{-5 - 7}{4 - (-2)} = \frac{-12}{6} = -2$ . The line falls 2 units for each 1 unit to the right.

**Special cases.** Two points with the same  $y$  give a horizontal line, rise 0, slope 0. Two points with the same  $x$  give a vertical line: the run is 0 and the slope is **undefined**, because we cannot divide by zero.

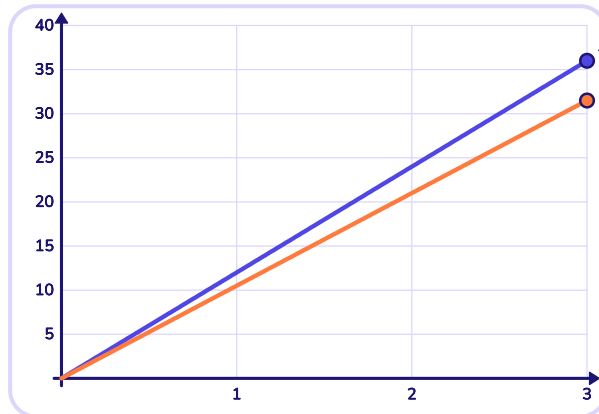
## 6. Comparing proportional relationships

To compare two relationships, find the unit rate of each, even when one is given as a table, one as a graph, and one as an equation. The larger unit rate is the faster rate, and its line is steeper.

## E

## Example 4: two cyclists

Cyclist A's graph passes through  $(0, 0)$  and  $(2, 24)$ , so her speed is  $\frac{24}{2} = 12$  miles per hour. Cyclist B follows  $d = 10.5t$ , so his speed is 10.5 miles per hour. Since  $12 > 10.5$ , cyclist A is faster, and her line is steeper.



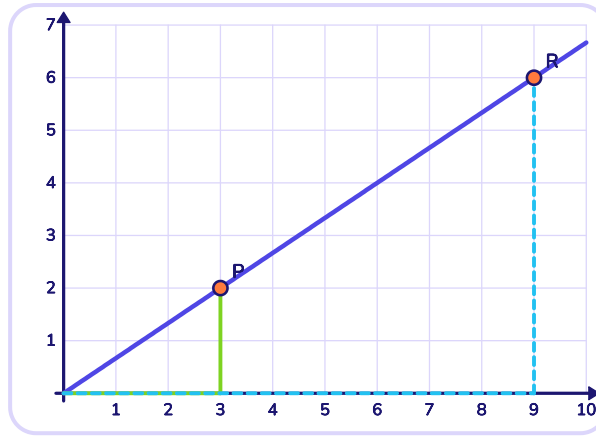
## 7. Similar triangles and slope

Why does a line have *one* slope, no matter which two points you pick? The reason is similar triangles. Draw a right triangle under the line with a horizontal leg (run) and a vertical leg (rise). A bigger triangle drawn the same way has the same angles, so the two triangles are **similar**, and their corresponding sides are in the same ratio.

## P

## Property

All slope triangles drawn on the same line are similar, so  $\frac{\text{rise}}{\text{run}}$  has the same value for every pair of points on that line.



In the figure, the small triangle has run 3 and rise 2; the large one has run 9 and rise 6. Both ratios simplify to  $\frac{2}{3}$ .

### E Example 5: using similar triangles

A zip line has a small slope triangle with run 5 feet and rise 2 feet. A larger triangle on the same cable has run 35 feet. The triangles are similar, so the rise is

$$35 \times \frac{2}{5} = 14 \text{ feet.}$$

## Key takeaways

- A relationship is proportional when  $\frac{y}{x}$  is constant; the constant is the unit rate  $k$ , and  $y = kx$ .
- Its graph is a straight line through the origin.
- Slope =  $\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$ ; for  $y = kx$  the slope is  $k$ .
- Up to the right means positive slope, down to the right means negative slope; horizontal is 0, vertical is undefined.
- To compare relationships, compare unit rates; the steeper line has the greater rate.
- Similar triangles explain why the slope is the same between any two points of a line.



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