

Probability and Statistics: math lesson, Grade 11 – download the PDF

How many different passwords can a phone have? How likely is it that a medical test is wrong? Can a poll of 1,000 people speak for a whole country? In this chapter you will learn to count outcomes quickly, compute probabilities of combined events, work with the bell-shaped normal curve, and judge how far a survey result can be trusted.

1. The counting principle and factorials

P

Fundamental counting principle

If a task is done in stages and stage 1 can happen in m_1 ways, stage 2 in m_2 ways, and so on up to stage k , then the whole task can be done in $m_1 \times m_2 \times \cdots \times m_k$ ways.

D

Factorial

For a whole number $n \geq 1$, $n! = n \times (n - 1) \times \cdots \times 2 \times 1$. By convention $0! = 1$.
For example $5! = 120$.

E Example 1: a password

A password has 3 different capital letters followed by 2 different digits. There are $26 \times 25 \times 24$ ways to choose the letters and 10×9 ways to choose the digits, so $26 \times 25 \times 24 \times 10 \times 9 = 1,404,000$ passwords.

2. Permutations and combinations

D Permutation

A permutation is an arrangement in which **order matters**. The number of ways to arrange r objects chosen from n different objects is $P(n, r) = \frac{n!}{(n-r)!}$.

D Combination

A combination is a selection in which **order does not matter**. The number of ways to choose r objects from n is $C(n, r) = \frac{n!}{r!(n-r)!}$. Always $C(n, r) = C(n, n-r)$.

M Method: permutation or combination?

1. Ask yourself: if I swap two chosen items, do I get a different result?
2. Yes (gold, silver, bronze medals): use $P(n, r)$.
3. No (a team, a hand of cards): use $C(n, r)$.

E**Example 2: medals and teams**

Eight runners race. Medals for first, second and third place:

$P(8, 3) = 8 \times 7 \times 6 = 336$. Choosing 3 of the 8 runners for a relay team:

$$C(8, 3) = \frac{336}{3!} = 56.$$

When some objects are identical, divide by the factorial of each repeated count. The letters of BANANA (B once, A three times, N twice) can be arranged in $\frac{6!}{3! 2!} = 60$ ways.

3. Independent and dependent events

D**Independent events**

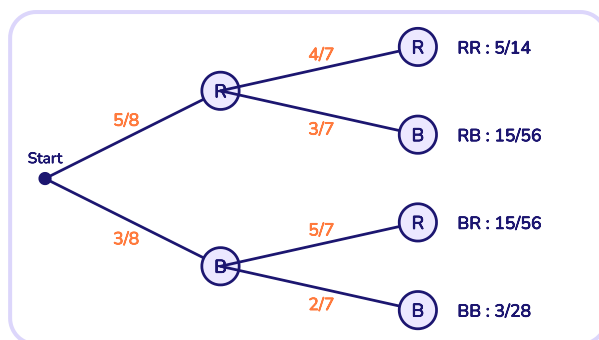
Events A and B are independent when one happening does not change the probability of the other. Then $P(A \text{ and } B) = P(A) \times P(B)$. If the second probability changes because of the first event, the events are **dependent**, and $P(A \text{ and } B) = P(A) \times P(B | A)$.

E**Example 3: drawing marbles**

A bag holds 5 red and 3 blue marbles. Two are drawn, and we want both red.

With replacement, the draws are independent: $\frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$.

Without replacement, the draws are dependent: $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$.



A tree diagram multiplies along each branch and adds the branches that give the same outcome.

The four leaves above add up to $\frac{20 + 15 + 15 + 6}{56} = 1$.

4. Conditional probability

D Conditional probability

The probability of B given that A has occurred is $P(B | A) = \frac{P(A \text{ and } B)}{P(A)}$, with $P(A) > 0$. Events are independent exactly when $P(B | A) = P(B)$.

A two-way table makes this concrete. In a survey of 200 students, we record band membership and honor roll status.

	On honor roll	Not on honor roll	Total
In band	48	32	80
Not in band	42	78	120
Total	90	110	200

E**Example 4: reading the table**

Given that a student is in band, the chance of being on the honor roll is

$P(H | B) = \frac{48}{80} = 0.6$. Overall $P(H) = \frac{90}{200} = 0.45$. Since $0.6 \neq 0.45$, the events are not independent.

A**Careful**

$P(B | A)$ and $P(A | B)$ are usually different. Here $P(B | H) = \frac{48}{90} = \frac{8}{15}$, not 0.6. Always check which group you are restricting to: it becomes the denominator.

5. Binomial distribution and the binomial theorem**D****Binomial experiment**

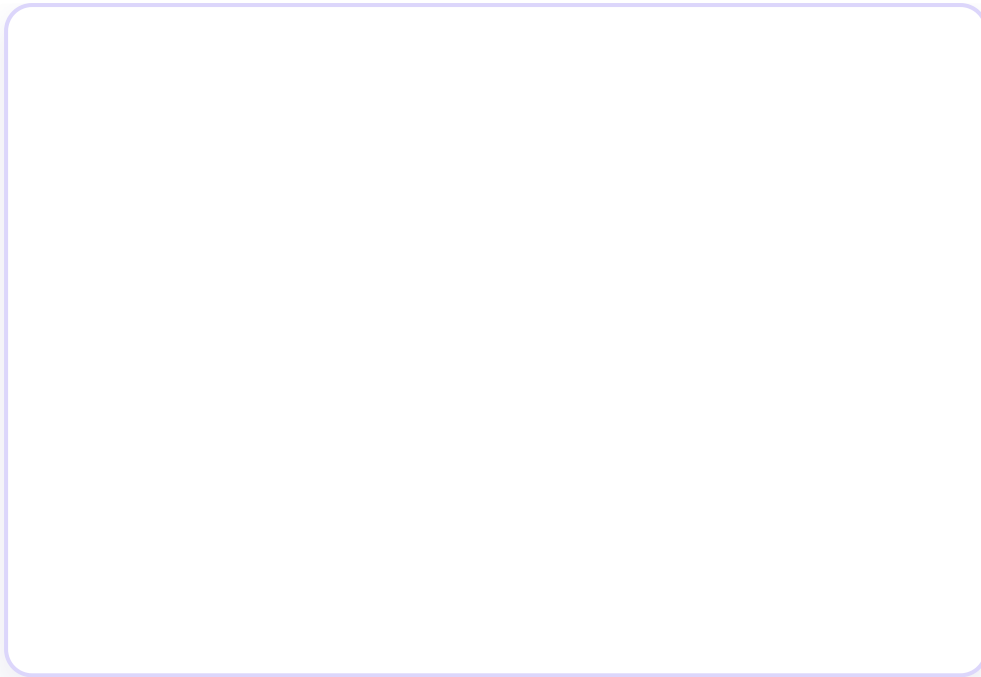
Repeat n independent trials, each with only two outcomes (success with probability p , failure with probability $1 - p$). The number of successes X satisfies

$P(X = k) = C(n, k) p^k (1 - p)^{n-k}$. Its mean is np and its standard deviation is $\sqrt{np(1 - p)}$.

E**Example 5: free throws**

A player makes 70% of free throws. In 5 shots,

$P(X = 3) = C(5, 3)(0.7)^3(0.3)^2 = 10 \times 0.343 \times 0.09 = 0.3087$. The mean is $5 \times 0.7 = 3.5$ made shots.



Probability and statistics grade 11: bar chart of the binomial distribution with n equal to 5 and p equal to 0.7, tallest bar at 4 successes

P Binomial theorem

$(a + b)^n = \sum_{k=0}^n C(n, k) a^{n-k} b^k$. The coefficients $C(n, k)$ are row n of Pascal's triangle.

E Example 6: expanding

Row 4 is 1, 4, 6, 4, 1. So

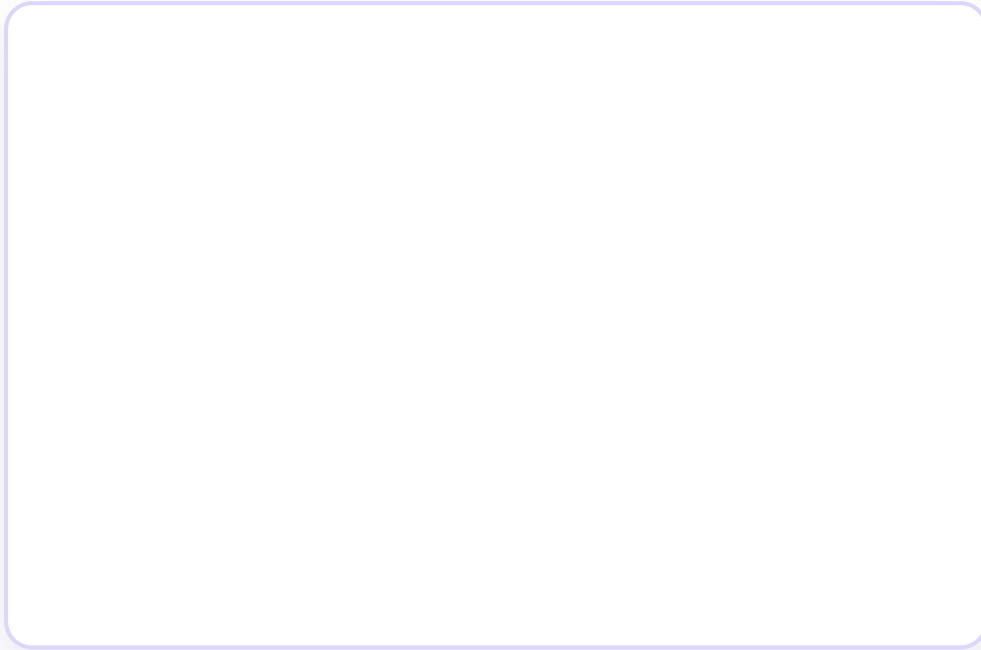
$$(x + 2)^4 = x^4 + 4 \cdot 2x^3 + 6 \cdot 4x^2 + 4 \cdot 8x + 16 = x^4 + 8x^3 + 24x^2 + 32x + 16.$$

6. The normal distribution and the empirical rule

Many measurements (heights, fill volumes, test scores) pile up around an average and thin out symmetrically on both sides. Their graph is a bell-shaped **normal curve** with mean μ and standard deviation σ .

P **Empirical rule (68-95-99.7)**

In a normal distribution, about 68% of the data lie within 1 standard deviation of the mean, about 95% within 2, and about 99.7% within 3.



Probability and statistics grade 11: normal bell curve shaded to show 68 percent within one standard deviation, 95 percent within two and 99.7 percent within three

E **Example 7: bottles**

A machine fills bottles with mean 16.0 oz and standard deviation 0.2 oz. About 95% of bottles hold between $16.0 - 0.4 = 15.6$ oz and $16.0 + 0.4 = 16.4$ oz (about 473 mL to 485 mL). The tails are symmetric, so about $\frac{100 - 95}{2} = 2.5\%$ of bottles hold more than 16.4 oz.

7. Standard deviation and z-scores

D Standard deviation

For a data set with mean $\bar{x}$ and n values, the population standard deviation is

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}. \text{ It measures the typical distance from the mean.}$$

M Method: computing σ

1. Find the mean.
2. Subtract the mean from each value and square the result.
3. Average the squares, then take the square root.

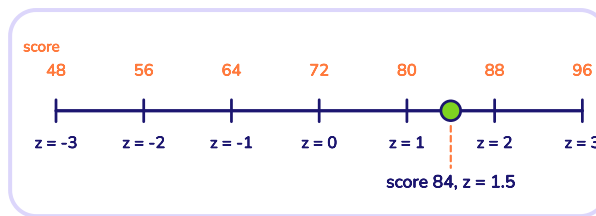
E Example 8: five numbers

For 4, 6, 8, 10, 12 the mean is 8. Squared deviations: **16, 4, 0, 4, 16**, sum **40**, so

$$\sigma = \sqrt{40/5} = \sqrt{8} \approx 2.83.$$

D z-score

The z-score of a value x is $z = \frac{x - \mu}{\sigma}$. It tells how many standard deviations x lies above ($z > 0$) or below ($z < 0$) the mean, which lets you compare different scales.



E Example 9: comparing scores

On a test with $\mu = 72$ and $\sigma = 8$, a score of 84 has $z = \frac{84 - 72}{8} = 1.5$.

8. Sampling methods and bias

A **population** is the whole group we care about; a **sample** is the part we actually survey. A good sample looks like a small copy of the population.

Method	How it works	Example
Simple random	Every member has an equal chance	Drawing 30 names from a hat
Stratified	Split into groups, sample each in proportion	10% of every grade level
Cluster	Pick whole groups at random	Surveying 4 randomly chosen classrooms
Systematic	Take every k-th member	Every 10th person on a list
Convenience	Whoever is easiest to reach	Asking people at one store (biased)



Bias

A sample is **biased** if some members of the population are more likely to be chosen than others. Convenience samples (asking people at a gym about exercise) and voluntary response samples (call-in polls) are usually biased, and a large sample size does not fix bias.

9. Margin of error and statistical inference

Inference means using a sample to draw a conclusion about a population. Two random samples give slightly different results, so every estimate comes with a **margin of error**. For a sample proportion $\hat{p}$ from a random sample of size n , an approximate 95% margin of error is

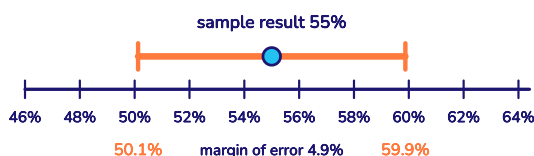
$$ME = 1.96 \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}, \text{ and the interval is } \hat{p} \pm ME.$$



Example 10: a poll

In a random sample of $n = 400$ voters, 55% support a park. Then

$ME = 1.96 \sqrt{\frac{0.55 \times 0.45}{400}} \approx 0.049$. We estimate that between about 50.1% and 59.9% of all voters support it.



Zyro's tip

On my planet we say: to cut the margin of error in half, you must survey four times as many beings. The square root of n is the boss!

Key takeaways

- Multiply the number of choices at each stage; $n!$ counts arrangements of n different objects.
- Order matters: $P(n, r) = \frac{n!}{(n-r)!}$. Order does not matter: $C(n, r) = \frac{n!}{r!(n-r)!}$.
- Independent: $P(A \text{ and } B) = P(A)P(B)$. In general $P(A \text{ and } B) = P(A)P(B | A)$.
- $P(B | A) = \frac{P(A \text{ and } B)}{P(A)}$.
- Binomial: $P(X = k) = C(n, k)p^k(1-p)^{n-k}$, mean np .
- Empirical rule: 68%, 95%, 99.7%. $z = \frac{x - \mu}{\sigma}$.
- Random samples avoid bias; $ME = 1.96\sqrt{\hat{p}(1-\hat{p})/n}$.



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