

Polynomial Functions: practice solutions, Grade 11 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Is it a polynomial?



- a. Yes. The exponents are whole numbers and $\sqrt{2}$ is just a constant coefficient.
- b. No. $\frac{4}{x} = 4x^{-1}$ has a negative exponent.
- c. No. $\sqrt{x} = x^{1/2}$ has a fractional exponent.
- d. Yes. $j(x) = \frac{1}{3}x^2 + \frac{1}{3}$, a quadratic.

2 Standard form and degree



Standard form: $f(x) = 7x^5 - 4x^3 - x + 9$.

The degree is **5**, the leading coefficient is **7** and the constant term is **9**.

3 Adding and subtracting



- a. Combine like terms: $3x^3 + 3x^2 - 2x - 6$.
- b. Distribute the minus sign: $6x^2 + x - 4 - 2x^2 + 3x - 9 = 4x^2 + 4x - 13$.

4 Multiplying binomials



(a) $x^2 - 3x + 5x - 15 = x^2 + 2x - 15$.

(b) $6x^2 + 8x - 3x - 4 = 6x^2 + 5x - 4$.

5 Evaluating with the Remainder Theorem



$p(2) = 8 - 8 + 10 - 1 = 9$ and $p(-1) = -1 - 2 - 5 - 1 = -9$.

By the Remainder Theorem, the remainder on division by $x - 2$ is **9** and the remainder on division by $x + 1$ is **-9**.

6 A first synthetic division



With $c = 2$ and coefficients **1, 2, -5, -6**: bring down **1**; $2 + 2 = 4$; $-5 + 8 = 3$; $-6 + 6 = 0$.

The quotient is $x^2 + 4x + 3$ and the remainder is **0**. Since the remainder is zero, $x - 2$ is a factor.

7 Cubes in two steps



(a) $x^3 + 4^3 = (x + 4)(x^2 - 4x + 16)$.

(b) $(2x)^3 - 1^3 = (2x - 1)(4x^2 + 2x + 1)$.

8 Reading end behavior



Look at the degree and the sign of the leading coefficient.

- a. Odd degree, negative leading coefficient: rises on the left, falls on the right.
- b. Even degree, positive leading coefficient: rises on both sides.
- c. Even degree, negative leading coefficient: falls on both sides.
- d. Odd degree, positive leading coefficient: falls on the left, rises on the right.

9 Long division with a remainder



$x^3 \div x = x^2$; $x^2(x - 2) = x^3 - 2x^2$; subtract: $-4x^2 + 11x$.

$-4x^2 \div x = -4x$; $-4x(x - 2) = -4x^2 + 8x$; subtract: $3x - 7$.

$3x \div x = 3$; $3(x - 2) = 3x - 6$; subtract: -1 .

So $x^3 - 6x^2 + 11x - 7 = (x - 2)(x^2 - 4x + 3) - 1$. The remainder is -1 , which agrees with $p(2) = 8 - 24 + 22 - 7 = -1$.

10 Missing terms



Write the dividend as $3x^3 + 0x^2 + 2x - 5$ and use $c = -1$.

Bring down 3; $0 - 3 = -3$; $2 + 3 = 5$; $-5 - 5 = -10$.

The quotient is $3x^2 - 3x + 5$ and the remainder is -10 .

11 Synthetic division, degree 4



The coefficients are 2, -5 , 0, -1 , 6 and $c = 2$. Bring down 2; $-5 + 4 = -1$; $0 - 2 = -2$; $-1 - 4 = -5$; $6 - 10 = -4$.

The quotient is $2x^3 - x^2 - 2x - 5$ and the remainder is -4 , so $p(2) = -4$.

12 Finding a missing coefficient



By the Factor Theorem, $p(-3) = 0$: $-27 + 9k + 12 - 12 = 0$, so $9k = 27$ and $k = 3$.

Then

$$p(x) = x^3 + 3x^2 - 4x - 12 = x^2(x + 3) - 4(x + 3) = (x + 3)(x^2 - 4) = (x + 3)(x - 2)$$

.

13 The packaging box



The volume is $x(x + 4)(x - 1) = x^3 + 3x^2 - 4x$, so $x^3 + 3x^2 - 4x - 42 = 0$.

Test $x = 3$: $27 + 27 - 12 - 42 = 0$. Synthetic division gives

$x^3 + 3x^2 - 4x - 42 = (x - 3)(x^2 + 6x + 14)$. The quadratic has discriminant $36 - 56 < 0$, so $x = 3$ is the only real solution.

The box is **3** in by **7** in by **2** in, which is about **7.6** cm by **17.8** cm by **5.1** cm. Check:
 $3 \times 7 \times 2 = 42$.

14 Rational root candidates



Divisors of 10: $\pm 1, \pm 2, \pm 5, \pm 10$. Divisors of 3: $\pm 1, \pm 3$. Candidates:

$$\pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}.$$

$p(1) = 3 - 1 + 8 - 10 = 0$. Synthetic division gives $3x^2 + 2x + 10$.

The discriminant is $4 - 120 = -116 < 0$, so $x = \frac{-2 \pm 2i\sqrt{29}}{6} = \frac{-1 \pm i\sqrt{29}}{3}$. The only real zero is **1**; the other two are non-real conjugates.

15 Sum and difference of cubes



(a) $(5x)^3 + 3^3 = (5x + 3)(25x^2 - 15x + 9)$.

(b) First factor out 2: $54x^3 - 2 = 2(27x^3 - 1) = 2(3x - 1)(9x^2 + 3x + 1)$.

16 Reading a graph



- a. The graph crosses the x-axis at -2 , 1 and 3 .
- b. With leading coefficient 1 : $k(x) = (x + 2)(x - 1)(x - 3)$. Check:
 $k(0) = 2 \times (-1) \times (-3) = 6$. Expanded, $k(x) = x^3 - 2x^2 - 5x + 6$.
- c. The degree is odd and the leading coefficient is positive, so the graph falls on the left and rises on the right.

17 Solving a cubic completely



Candidates: $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$. Try $x = -2$: $-16 - 4 + 26 - 6 = 0$.

Synthetic division with $c = -2$: $2; -1 - 4 = -5; -13 + 10 = -3; -6 + 6 = 0$. The quotient is $2x^2 - 5x - 3 = (2x + 1)(x - 3)$.

So $(x + 2)(2x + 1)(x - 3) = 0$ and the solutions are $-2, -\frac{1}{2}$ and 3 .

18 Building a polynomial from its zeros



Real coefficients force the conjugate $1 - 3i$ to be a zero too. Then

$$(x - (1 + 3i))(x - (1 - 3i)) = (x - 1)^2 - (3i)^2 = x^2 - 2x + 10.$$

Multiply by $(x - 2)$:

$$(x - 2)(x^2 - 2x + 10) = x^3 - 2x^2 + 10x - 2x^2 + 4x - 20 = x^3 - 4x^2 + 14x - 20.$$

The degree is 3 , matching the three zeros $2, 1 + 3i, 1 - 3i$.

19 Six zeros

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$x^6 - 64 = (x^2)^3 - 4^3 = (x^2 - 4)(x^4 + 4x^2 + 16)$; instead split as $(x^3 - 8)(x^3 + 8)$.

$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$ and $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$.

The linear factors give **2** and **-2**. The quadratics give $x = \frac{-2 \pm \sqrt{4 - 16}}{2} = -1 \pm i\sqrt{3}$ and $x = 1 \pm i\sqrt{3}$.

Six zeros for degree **6**: ± 2 , $-1 \pm i\sqrt{3}$, $1 \pm i\sqrt{3}$.

20 Two conditions, two unknowns

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$p(1) = 1 + a + b - 6 = -8$ gives $a + b = -3$. $p(-1) = -1 + a - b - 6 = 0$ gives $a - b = 7$.

Adding the equations: $2a = 4$, so $a = 2$ and $b = -5$.

Then $p(x) = x^3 + 2x^2 - 5x - 6 = (x + 1)(x^2 + x - 6) = (x + 1)(x + 3)(x - 2)$.

21 True or false?

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- a. True. The two ends go in opposite directions, so the graph must cross the x-axis somewhere.
- b. True. For example, $x^4 + 1$ is never **0** for real x ; its four zeros are all non-real.
- c. False. The theorem counts **5** zeros with multiplicity, in the complex numbers. For instance x^5 has the single real zero **0** with multiplicity **5**.

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