

Parallel and Perpendicular Lines: math lesson, Grade 10 – download the PDF

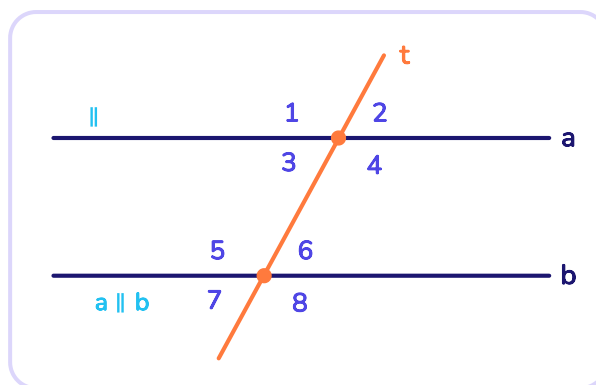
Railroad tracks never meet, a crosswalk meets the curb at a square corner, and a ruler slid along a triangle draws lines that stay the same distance apart. In this chapter you will learn how angles, slopes and distances let you prove that lines are parallel or perpendicular, write their equations, and measure the shortest way from a point to a line.

1. Transversals and angle pairs

D Definition: transversal

A **transversal** is a line that crosses two or more other lines in the same plane at different points. Two crossing points give **eight angles**.

Look at the figure. Lines ***a*** and ***b*** are cut by the transversal ***t***. The angles are numbered 1 to 8: angles 1 to 4 sit around the upper crossing, angles 5 to 8 around the lower one, each time in the order upper left, upper right, lower left, lower right.



Angles are named by their position, and each name comes with a picture in your head:

Pair type	Position	If the lines are parallel
Corresponding	same corner at each crossing	congruent
Alternate interior	between the lines, opposite sides of t	congruent
Alternate exterior	outside the lines, opposite sides of t	congruent
Same-side interior	between the lines, same side of t	supplementary (sum 180°)

In the figure, $\angle 2$ and $\angle 6$ are corresponding, $\angle 3$ and $\angle 6$ are alternate interior, $\angle 4$ and $\angle 6$ are same-side interior, and $\angle 1$ and $\angle 8$ are alternate exterior. Two angles that share a crossing point also have simple relationships: **vertical angles** such as $\angle 2$ and $\angle 3$ are always congruent, and a **linear pair** such as $\angle 1$ and $\angle 2$ always adds up to 180° .

2. Angles formed by parallel lines

P Theorem: parallel lines and angle pairs

If two **parallel** lines are cut by a transversal, then:

- corresponding angles are congruent;
- alternate interior angles are congruent;
- alternate exterior angles are congruent;
- same-side interior angles are supplementary: $m\angle 3 + m\angle 5 = 180^\circ$.

You only need to remember one fact by heart, the corresponding angles postulate. The other three follow from it with vertical angles and linear pairs. For example, $\angle 3 \cong \angle 7$ (corresponding) and $\angle 7 \cong \angle 6$ (vertical), so $\angle 3 \cong \angle 6$, which is the alternate interior theorem.

E Example 1: solving for x

Parallel lines are cut by a transversal. Two corresponding angles measure $(3x + 10)^\circ$ and $(5x - 30)^\circ$. Find x and the angle measure.

Corresponding angles are congruent, so $3x + 10 = 5x - 30$. Then $40 = 2x$ and $x = 20$. Each angle measures $3(20) + 10 = 70^\circ$. Check: $5(20) - 30 = 70$. The angles measure 70° .



Common mistake

These angle facts only work **when the lines are parallel**. If the lines are not known to be parallel, congruent alternate interior angles are something you have to prove, not something you may assume.

3. Proving that lines are parallel



Converse theorems

If two lines are cut by a transversal and any one of these is true, then the lines are parallel:

- a pair of corresponding angles is congruent;
- a pair of alternate interior (or alternate exterior) angles is congruent;
- a pair of same-side interior angles is supplementary.

Also, two lines that are both perpendicular to the same line are parallel to each other.



Method: prove two lines are parallel

1. Mark the transversal and identify the pair of angles you know.
2. Name the pair (corresponding, alternate interior, same-side interior...).
3. Check the condition: equal measures, or a sum of **180°** for same-side interior angles.
4. Conclude with the matching converse theorem.

E**Example 2: are they parallel?**

A transversal makes same-side interior angles of 112° and 68° . Since $112 + 68 = 180$, the angles are supplementary, so the two lines are parallel by the converse of the same-side interior angle theorem. If the angles had been 112° and 65° , the sum 177° would not equal 180° and the lines would not be parallel.

4. Slopes of parallel and perpendicular lines

In a coordinate plane the slope of a line through (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

P**Slope criteria**

Two non-vertical lines with slopes m_1 and m_2 are:

- **parallel** if and only if $m_1 = m_2$ (and they are different lines);
- **perpendicular** if and only if $m_1 \cdot m_2 = -1$, that is, $m_2 = -\frac{1}{m_1}$. Their slopes are **opposite reciprocals**.

A horizontal line (slope 0) is perpendicular to a vertical line (undefined slope).

Why -1 ? Turning the run-rise triangle of a line a quarter turn swaps the roles of run and rise and changes one sign: a slope of $\frac{3}{5}$ becomes $-\frac{5}{3}$.



Common mistake

Flipping the fraction is not enough, and changing only the sign is not enough either.

The perpendicular slope to $\frac{2}{7}$ is $-\frac{7}{2}$: you need **both** the flip and the sign change.

5. Equations of parallel and perpendicular lines



Method: line through a point

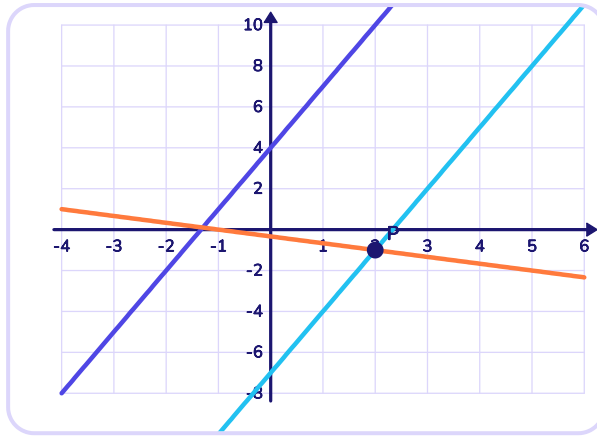
1. Write the given line in slope-intercept form $y = mx + b$ to read its slope m .
2. Choose the new slope: m for a parallel line, $-\frac{1}{m}$ for a perpendicular line.
3. Use point-slope form with the given point (x_0, y_0) : $y - y_0 = m_{\text{new}}(x - x_0)$.
4. Simplify, then check that the point satisfies your equation.



Example 3: parallel and perpendicular through the same point

Line ℓ : $y = 3x + 4$. Point $P(2, -1)$.

Parallel: slope 3, so $y + 1 = 3(x - 2)$, which gives $y = 3x - 7$. Perpendicular: slope $-\frac{1}{3}$, so $y + 1 = -\frac{1}{3}(x - 2)$, which gives $y = -\frac{1}{3}x - \frac{1}{3}$. Check with $x = 2$: $3(2) - 7 = -1$ and $-\frac{2}{3} - \frac{1}{3} = -1$. Both lines pass through P .



Zyro's tip

On my home planet we say: “same slope, never meet; opposite reciprocal, meet square.” Multiply the two slopes: you get -1 for perpendicular lines every time!

6. Distance from a point to a line



Definition: distance from a point to a line

The distance from a point P to a line ℓ is the length of the **perpendicular segment** from P to ℓ . It is the shortest segment joining P to the line.



Distance formula

For the line $Ax + By + C = 0$ and the point $P(x_0, y_0)$:

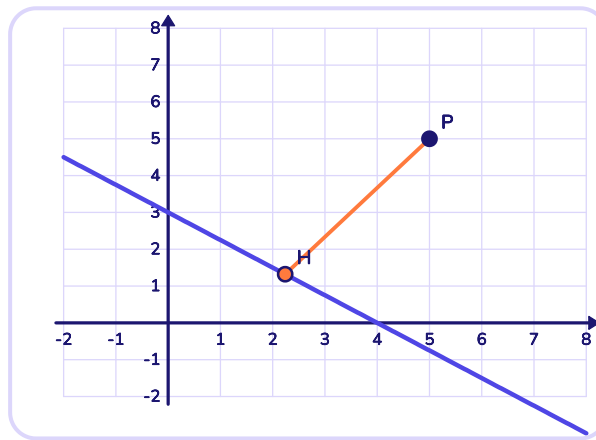
$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

E**Example 4: distance with the formula**

Find the distance from $P(5, 5)$ to the line $3x + 4y = 12$. Rewrite it as

$$3x + 4y - 12 = 0. \text{ Then } d = \frac{|3(5) + 4(5) - 12|}{\sqrt{3^2 + 4^2}} = \frac{23}{5} = 4.6 \text{ units.}$$

Without the formula: the perpendicular through P has slope $\frac{4}{3}$; it meets the line at $H(2.24, 1.32)$, and $PH = \sqrt{2.76^2 + 3.68^2} = \sqrt{21.16} = 4.6$.



To find the distance between two **parallel lines**, pick any point on one line and measure its distance to the other.

7. The perpendicular bisector theorem

D**Definition: perpendicular bisector**

The **perpendicular bisector** of a segment $\overline{AB}$ is the line that is perpendicular to $\overline{AB}$ and passes through its midpoint.

P**Perpendicular bisector theorem and its converse**

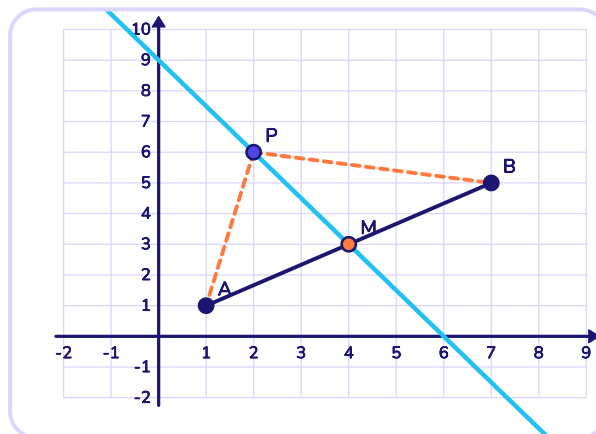
A point is on the perpendicular bisector of $\overline{AB}$ if and only if it is equidistant from A and B : $PA = PB$.

E**Example 5: building a perpendicular bisector**

Let $A(1, 1)$ and $B(7, 5)$. The midpoint is $M\left(\frac{1+7}{2}, \frac{1+5}{2}\right) = (4, 3)$. The slope of $\overline{AB}$ is $\frac{5-1}{7-1} = \frac{2}{3}$, so the bisector has slope $-\frac{3}{2}$. Then $y - 3 = -\frac{3}{2}(x - 4)$, so $y = -\frac{3}{2}x + 9$.

Test the theorem with $P(2, 6)$, which is on the line since $-3 + 9 = 6$:

$PA = \sqrt{1^2 + 5^2} = \sqrt{26}$ and $PB = \sqrt{5^2 + 1^2} = \sqrt{26}$. They are equal, as predicted.



Key takeaways

- A transversal creates corresponding, alternate interior, alternate exterior and same-side interior angle pairs.
- With parallel lines, the first three pairs are congruent and same-side interior angles add up to 180° . The converses prove lines parallel.
- Parallel lines have equal slopes; perpendicular lines have slopes whose product is -1 .
- Use point-slope form $y - y_0 = m(x - x_0)$ to write a line through a given point.
- Distance from $P(x_0, y_0)$ to $Ax + By + C = 0$: $d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$.
- A point is on the perpendicular bisector of $\overline{AB}$ exactly when $PA = PB$.



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