

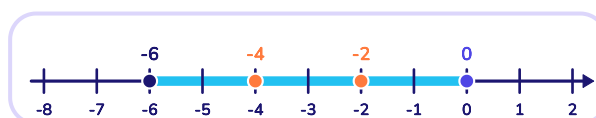
Multiplying and Dividing Rational Numbers: math lesson, Grade 7 – download the PDF

Rational numbers are all the numbers you can write as a fraction of two integers: -4 , $\frac{3}{5}$, $2\frac{1}{3}$, -0.75 . In this chapter you will learn to multiply and divide them, to turn fractions into decimals (some of which never end!), and to combine all these skills in longer calculations. Let's go.

1. Multiplying integers and the sign rules

Multiplication is repeated addition. Adding three copies of -2 gives

$3 \cdot (-2) = (-2) + (-2) + (-2) = -6$. On a number line this means three jumps of 2 units to the left, starting at 0.



P**Sign rules for multiplication**

The absolute value of a product is the product of the absolute values. The sign depends only on the signs of the factors:

- same signs: the product is positive;
- different signs: the product is negative;
- a product with zero as a factor is 0.

Factors	Signs	Product
$6 \cdot 4$	positive, positive	positive: 24
$(-6) \cdot 4$	negative, positive	negative: -24
$6 \cdot (-4)$	positive, negative	negative: -24
$(-6) \cdot (-4)$	negative, negative	positive: 24

Why is a negative times a negative positive? Look at the pattern $3 \cdot (-4) = -12$, $2 \cdot (-4) = -8$, $1 \cdot (-4) = -4$, $0 \cdot (-4) = 0$. The products go up by 4 each time, so the next one must be $(-1) \cdot (-4) = 4$.

M**Multiplying several integers**

1. Count the negative factors.
2. An even count gives a positive product; an odd count gives a negative product.
3. Multiply the absolute values.

E**Example 1**

Compute $(-4)(-7)(-2)$. There are three negative factors (odd), so the result is negative. Since $4 \cdot 7 \cdot 2 = 56$, the product is -56 .

2. Dividing integers

Division undoes multiplication. Because $(-6) \cdot 4 = -24$, we know $-24 \div 4 = -6$ and $-24 \div (-6) = 4$. Every multiplication fact gives a family of division facts, so the sign rules are the same:

P**Sign rules for division**

Same signs give a positive quotient. Different signs give a negative quotient.

$0 \div a = 0$ for any $a \neq 0$, and you can never divide by 0.

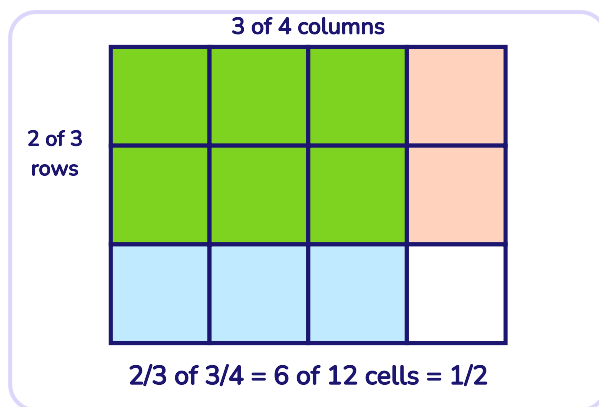
Examples: $-56 \div (-7) = 8$, $45 \div (-9) = -5$, $0 \div (-13) = 0$. A quotient can also be written as a fraction: $\frac{-12}{4} = -3$, and $\frac{12}{-4} = \frac{-12}{4} = -\frac{12}{4}$ are all the same number.

**Common mistake**

$-8 - 3$ is a subtraction, not a product. The sign rules for multiplication do not apply here: $-8 - 3 = -11$, whereas $(-8) \cdot (-3) = 24$.

3. Multiplying fractions and mixed numbers

To multiply fractions, multiply the numerators and multiply the denominators. The area model shows why: taking $\frac{2}{3}$ of $\frac{3}{4}$ means shading 2 of 3 rows inside 3 of 4 columns.



The overlap has 6 cells out of 12, so $\frac{2}{3} \cdot \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$.

M

Multiplying fractions and mixed numbers

1. Rewrite each mixed number as an improper fraction.
2. Decide the sign with the sign rules.
3. Cancel common factors between any numerator and any denominator.
4. Multiply what is left and write the answer in lowest terms (as a mixed number if you like).

E

Example 2

Compute $-\frac{3}{4} \cdot \frac{8}{9}$. The signs differ, so the result is negative. Cancel 3 with 9 and 4 with 8:

$$-\frac{3}{4} \cdot \frac{8}{9} = -\frac{1 \cdot 2}{1 \cdot 3} = -\frac{2}{3}.$$

4. Dividing fractions and mixed numbers

How many halves fit in 3? Six, because each whole holds two halves. So $3 \div \frac{1}{2} = 6 = 3 \cdot 2$.



$3 \div (1/2) = 6$, because 6 halves fit in 3 wholes

D Reciprocal

The reciprocal of a nonzero number $\frac{a}{b}$ is $\frac{b}{a}$. A number times its reciprocal equals 1, and both have the same sign. For example, the reciprocal of -5 is $-\frac{1}{5}$.

P Dividing by a fraction

Dividing by a nonzero rational number is the same as multiplying by its reciprocal:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}.$$

E Example 3

Compute $2\frac{1}{3} \div \left(-1\frac{1}{6}\right)$. First, $2\frac{1}{3} = \frac{7}{3}$ and $1\frac{1}{6} = \frac{7}{6}$. Then

$$\frac{7}{3} \div \left(-\frac{7}{6}\right) = \frac{7}{3} \cdot \left(-\frac{6}{7}\right) = -\frac{42}{21} = -2.$$

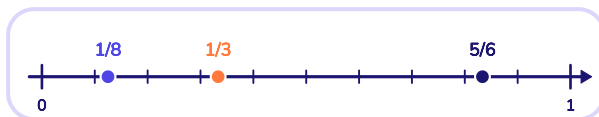


Common mistake

Flip only the second fraction, the divisor. Never flip the first one, and never flip while multiplying.

5. Converting fractions to decimals

A fraction is a division: $\frac{3}{8}$ means $3 \div 8$. Use long division and keep going until the remainder is 0 or a remainder repeats. Then the decimal either terminates or repeats forever.



Fraction	Decimal	Type
$\frac{1}{8}$	0.125	terminating
$\frac{1}{3}$	$0.\overline{3}$	repeating
$\frac{5}{6}$	$0.8\overline{3}$	repeating
$\frac{4}{11}$	$0.\overline{36}$	repeating
$\frac{3}{7}$	$0.\overline{428571}$	repeating

The bar marks the digits that repeat. A repeating block can be at most as long as the denominator minus 1, since only that many different nonzero remainders exist.

P Terminating or repeating?

Write the fraction in lowest terms. If the denominator has no prime factors other than 2 and 5, the decimal terminates. Otherwise it repeats.

E**Example 4**

Convert $\frac{5}{11}$. Divide: $5.000 \dots \div 11$ gives remainders 6, 5, 6, 5, ... so the digits **4, 5** repeat: $\frac{5}{11} = 0.\overline{45}$. Because **11** is not made of 2s and 5s, we expected a repeating decimal.

6. Order of operations with rational numbers

The order of operations (PEMDAS) works exactly as with whole numbers: Parentheses, Exponents, then Multiplication and Division from left to right, and finally Addition and Subtraction from left to right. Multiplication and division share the same rank, so you never do all multiplications first.

E**Example 5**

Compute $\frac{1}{2} + \left(-\frac{3}{4}\right) \div \left(-\frac{3}{2}\right)$. Do the division first:
 $\left(-\frac{3}{4}\right) \cdot \left(-\frac{2}{3}\right) = \frac{6}{12} = \frac{1}{2}$. Then $\frac{1}{2} + \frac{1}{2} = 1$.

!**Zyro's tip**

On my planet we read left to right, just like in this rule. When a long expression scares you, rewrite it one line at a time and change only one operation per line.

7. Properties of operations

These properties hold for all rational numbers and let you calculate in a smarter order.

- **Commutative:** $a \cdot b = b \cdot a$.

- **Associative:** $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
- **Identity:** $a \cdot 1 = a$.
- **Inverse:** $a \cdot \frac{1}{a} = 1$ for $a \neq 0$.
- **Distributive:** $a(b + c) = ab + ac$.

E

Example 6

Compute $12 \left(-\frac{2}{3} + \frac{3}{4} \right)$ the smart way. By the distributive property,

$$12 \cdot \left(-\frac{2}{3} \right) + 12 \cdot \frac{3}{4} = -8 + 9 = 1.$$

A

Common mistake

Division is not commutative: $6 \div 2 = 3$ but $2 \div 6 = \frac{1}{3}$. The same holds for subtraction.

8. Multistep problems

Real problems mix several operations. Follow a plan: read, choose the unknown, translate to an expression, compute in the right order, check that the answer makes sense, and write a sentence with units.

E

Example 7

A glacier's edge moves -2.5 inches each day for 6 days. Then it moves $+1\frac{1}{2}$ inches. Total change: $6 \cdot (-2.5) + 1.5 = -15 + 1.5 = -13.5$. The edge moved back 13.5 inches (about 34.3 cm).

Key takeaways

- Same signs give a positive product or quotient; different signs give a negative one.
- An even number of negative factors makes the product positive.
- Multiply fractions straight across; convert mixed numbers to improper fractions first.
- Dividing by a fraction means multiplying by its reciprocal.
- Every fraction is a terminating or a repeating decimal; the denominator in lowest terms tells which.
- Follow PEMDAS, working left to right for multiplication and division.
- Use the distributive, commutative and associative properties to simplify.



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