

Matrices and Systems of Equations: practice solutions, Grade 12 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Reading a matrix



(a) M has 2 rows and 3 columns, so its size is 2×3 .

(b) m_{12} is in row 1, column 2: $m_{12} = -2$. m_{23} is in row 2, column 3: $m_{23} = 9$.

2 Adding and subtracting



Combine entries in the same position.

$$P + Q = \begin{bmatrix} 5 & 3 \\ -6 & 6 \end{bmatrix} \text{ and } P - Q = \begin{bmatrix} 1 & -5 \\ 6 & 4 \end{bmatrix}.$$

3 Scalar multiples



Multiply every entry by the scalar.

$$(a) -2R = \begin{bmatrix} -2 & 6 & -4 \\ 0 & -8 & 10 \end{bmatrix}.$$

$$(b) \frac{1}{2}S = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}.$$

4 Is the product defined?



The inner sizes must match.

- a. AB : $(2 \times 3)(3 \times 4)$, inner numbers match, so the size is 2×4 .
- b. BA : $(3 \times 4)(2 \times 3)$, and $4 \neq 2$, so it is not defined.
- c. CD : $(3 \times 1)(1 \times 3)$ gives a 3×3 matrix.
- d. DC : $(1 \times 3)(3 \times 1)$ gives a 1×1 matrix.

5 Two-by-two determinants



Use $ad - bc$.

- a. $5(4) - 2(3) = 20 - 6 = 14$.
- b. $(-3)(-2) - 6(1) = 6 - 6 = 0$.
- c. $0(4) - 7(-1) = 7$.

The matrix in (b) has determinant 0 , so it has no inverse.

6 Check an inverse



Multiply: $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 6 - 5 & -3 + 3 \\ 10 - 10 & -5 + 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

The product is the identity matrix I , so the matrices are inverses. (Also $\det A = 6 - 5 = 1$.)

7 Write a system as a matrix equation



$$\begin{bmatrix} 4 & 3 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ -1 \end{bmatrix}.$$

Substitute $x = 2$, $y = 1$: $4(2) + 3(1) = 11$ and $2 - 3(1) = -1$. Both equations hold, so $(2, 1)$ is a solution.

8 Multiplying in both orders



$$AB = \begin{bmatrix} 1(5) + 4(2) & 1(0) + 4(-1) \\ -2(5) + 3(2) & -2(0) + 3(-1) \end{bmatrix} = \begin{bmatrix} 13 & -4 \\ -4 & -3 \end{bmatrix}.$$

$$BA = \begin{bmatrix} 5(1) + 0(-2) & 5(4) + 0(3) \\ 2(1) + (-1)(-2) & 2(4) + (-1)(3) \end{bmatrix} = \begin{bmatrix} 5 & 20 \\ 4 & 5 \end{bmatrix}.$$

$AB \neq BA$: matrix multiplication is not commutative.

9 Store revenue



$$\begin{bmatrix} 30 & 12 & 20 \\ 25 & 18 & 10 \end{bmatrix} \begin{bmatrix} 0.50 \\ 3.20 \\ 2.40 \end{bmatrix} = \begin{bmatrix} 101.40 \\ 94.10 \end{bmatrix}.$$

East: $30(0.5) + 12(3.2) + 20(2.4) = 15 + 38.4 + 48 = 101.4$. West:

$12.5 + 57.6 + 24 = 94.1$.

The East store earned \$101.40 and the West store earned \$94.10.

10 A three-by-three determinant



Expand along the first row: $1((-1)(5) - 4(0)) - 2(3(5) - 4(2)) + 0$.

$= 1(-5) - 2(7) = -5 - 14 = -19$.

11 Finding inverses



a. $\det = 16 - 15 = 1$, so the inverse is $\begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$.

b. $\det = 8 - 6 = 2$, so the inverse is $\frac{1}{2} \begin{bmatrix} 4 & -6 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -\frac{1}{2} & 1 \end{bmatrix}$. Check: the first

row of the product is $(4 - 3, -6 + 6) = (1, 0)$ and the second row is

$(2 - 2, -3 + 4) = (0, 1)$.

12 Solve with an inverse



$$A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}, \det A = 5 - 6 = -1, \text{ so } A^{-1} = -1 \cdot \begin{bmatrix} 1 & -3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}.$$

$$X = A^{-1}C = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} 11 \\ 5 \end{bmatrix} = \begin{bmatrix} -11 + 15 \\ 22 - 25 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}.$$

The solution is $x = 4, y = -3$. Check: $20 - 9 = 11$ and $8 - 3 = 5$.

13 Gaussian elimination



Augmented matrix rows: $(1, 2, -1 \mid 3), (2, 3, 1 \mid 11), (3, -1, 1 \mid 10)$.

$$R_2 - 2R_1 = (0, -1, 3 \mid 5) \text{ and } R_3 - 3R_1 = (0, -7, 4 \mid 1).$$

$R_3 - 7R_2 = (0, 0, -17 \mid -34)$, so $z = 2$. Then $-y + 6 = 5$ gives $y = 1$, and $x = 3 - 2 + 2 = 3$.

The solution is $(3, 1, 2)$.

14 Cramer's Rule practice



$$D = 2(-3) - 5(4) = -26.$$

$$D_x = (-4)(-3) - 5(18) = 12 - 90 = -78, \text{ so } x = \frac{-78}{-26} = 3.$$

$$D_y = 2(18) - (-4)(4) = 36 + 16 = 52, \text{ so } y = \frac{52}{-26} = -2.$$

The solution is $(3, -2)$.

15 Shear then reflection

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- (a) The shear acts first, so the combined matrix is $FH = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$.
- (b) $(1, 1) \rightarrow (1 + 2, -1) = (3, -1)$; $(4, 1) \rightarrow (6, -1)$; $(1, 3) \rightarrow (1 + 6, -3) = (7, -3)$.
- (c) T has base **3** and height **2**, so its area is **3**. Since $\det(FH) = -1$, the image area is $|-1| \cdot 3 = 3$. Check: the image has a horizontal base of length **3** and height **2**, area **3**.

16 Rotation and reflection: order matters

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$$FR = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \text{ (rotate first, then reflect).}$$

$$RF = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ (reflect first, then rotate).}$$

FR sends $(2, 5)$ to $(-5, -2)$, while RF sends it to $(5, 2)$. The results differ, so the order of the transformations matters.

17 Area with determinants

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(a) The area is $\frac{1}{2}|5(4) - 1(2)| = \frac{18}{2} = 9$.

(b) $P' = (3(5) + 0, 1(5) + 2(1)) = (15, 7)$ and $Q' = (6, 2 + 8) = (6, 10)$. The area is $\frac{1}{2}|15(10) - 7(6)| = \frac{108}{2} = 54$.

(c) $\det M = 3(2) - 0(1) = 6$, and $54 = 6 \times 9$: the transformation multiplies areas by $|\det M|$.

18 A system with a parameter

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(a) $D = 2(9) - k(6) = 18 - 6k = 0$ gives $k = 3$.

(b) With $k = 3$, multiplying $2x + 3y = 5$ by 3 gives $6x + 9y = 15$, which contradicts $6x + 9y = 12$. The lines are parallel, so there is **no solution**.

(c) For $k = 1$: $D = 18 - 6 = 12$, $D_x = 5(9) - 1(12) = 33$, $D_y = 2(12) - 5(6) = -6$.

So $x = \frac{33}{12} = \frac{11}{4}$ and $y = \frac{-6}{12} = -\frac{1}{2}$.

19 Trail mix prices

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Equations: $2a + r + 3c = 40$, $a + 2r + c = 22$, $3a + r + 2c = 38$. Swap rows 1 and 2 to get a leading 1: $(1, 2, 1 \mid 22)$, $(2, 1, 3 \mid 40)$, $(3, 1, 2 \mid 38)$.

$R_2 - 2R_1 = (0, -3, 1 \mid -4)$ and $R_3 - 3R_1 = (0, -5, -1 \mid -28)$.

$3R_3 - 5R_2 = (0, 0, -8 \mid -64)$, so $c = 8$. Then $-3r + 8 = -4$ gives $r = 4$, and $a = 22 - 8 - 8 = 6$.

Almonds cost \$6 per pound, raisins \$4, and cashews \$8.

20 True or false: $AB = 0$

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False. Take $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. Neither is the zero matrix, but

$$AB = \begin{bmatrix} 1-1 & -1+1 \\ 1-1 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Both matrices have determinant 0, which is why this can happen: matrices without inverses can multiply to zero, unlike ordinary numbers.



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