

Logarithmic Functions: practice solutions, Grade 11 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Logarithmic form



Use $b^y = x \iff \log_b x = y$: the base stays the base, and the exponent becomes the value of the logarithm.

a. $\log_2 64 = 6$

b. $\log_{10} 0.01 = -2$

c. $\log_7 49 = 2$

d. $\log_5 1 = 0$

e. $\log_9 3 = \frac{1}{2}$

2 Exponential form



a. $3^4 = 81$

b. $5^3 = 125$

c. $2^{-3} = \frac{1}{8}$

d. $16^{1/2} = 4$

e. $10^3 = 1000$ (the base of **log** is 10)

3 Evaluating logarithms



a. $2^4 = 16$, so the value is 4.

b. $5^{-2} = \frac{1}{25}$, so -2 .

c. $7^1 = 7$, so 1.

d. $9^0 = 1$, so 0.

e. $4^{1/2} = 2$, so $\frac{1}{2}$.

f. $\left(\frac{1}{2}\right)^{-3} = 8$, so -3 .

4 True or false?



a. **False.** $\log_2 0$ is undefined: no power of 2 equals 0. (Note $\log_2 1 = 0$.)

b. **True.** A power of 3 is never negative.

c. **True.** $\log_b b^x = x$.

d. **True.** $10^2 = 100$.

e. **False.** $\log_2 8 + \log_2 4 = 3 + 2 = 5 = \log_2 32$, and $\log_2 12 \approx 3.58$.

5 Domain of a logarithm



The expression inside the logarithm must be positive.

a. $x - 4 > 0$, so $x > 4$.

b. $2x + 10 > 0$, so $x > -5$.

c. $8 - x > 0$, so $x < 8$.

6 Splitting products



- a. $\log_2 8 + \log_2 x = 3 + \log_2 x$.
- b. $\log 1000 + \log y^2 = 3 + 2 \log y$.
- c. $\log_5 x^3 - \log_5 25 = 3 \log_5 x - 2$.

7 Natural logarithm values



- a. $\ln e^4 = 4$.
- b. $e^{\ln 9} = 9$.
- c. $\ln 1 = 0$.
- d. $\ln e^{-1} = -1$.
- e. $\ln e^{1/2} = \frac{1}{2}$.

8 Condensing logarithms



- a. $\log_3(4 \cdot 5) = \log_3 20$.
- b. $\log_2 \frac{48}{3} = \log_2 16 = 4$.
- c. $\log 25 + \log 4 = \log 100 = 2$.
- d. $\ln 81^{1/2} - \ln 3 = \ln 9 - \ln 3 = \ln 3$.

9 Change of base



a. $\frac{\ln 20}{\ln 3} \approx \frac{2.9957}{1.0986} \approx 2.727.$

b. $\frac{\ln 50}{\ln 7} \approx \frac{3.9120}{1.9459} \approx 2.010.$

c. $\frac{\ln 10}{\ln 0.5} \approx \frac{2.3026}{-0.6931} \approx -3.322.$ The result is negative because the base is less than 1 and 10 is greater than 1.

10 Same base



a. $5^x = 5^3$, so $x = 3.$

b. $4^{x+1} = 4^3$, so $x + 1 = 3$ and $x = 2.$

c. $\frac{1}{32} = 2^{-5}$, so $3x - 1 = -5$, $3x = -4$, $x = -\frac{4}{3}.$

d. $9^x = 3^{2x}$ and $27 = 3^3$, so $2x = 3$ and $x = \frac{3}{2}.$

11 Simple logarithmic equations



a. $x + 3 = 2^5 = 32$, so $x = 29.$ Check: $x + 3 = 32 > 0.$

b. $2x - 1 = 25$, so $x = 13.$

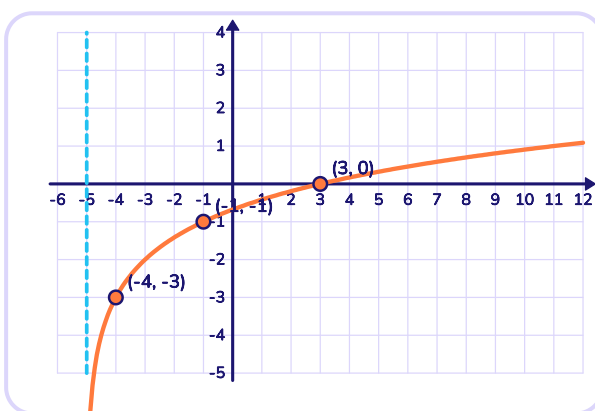
c. $x = 10^{-2} = 0.01.$

d. $x = e^3 \approx 20.09.$

12 Reading a logarithmic graph



- a. We need $x + 5 > 0$: the domain is $x > -5$ and the asymptote is $x = -5$ (the parent graph shifted 5 units left and 3 units down).
- b. $f(x) = 0 \iff \log_2(x + 5) = 3 \iff x + 5 = 8$, so $x = 3$. The x-intercept is $(3, 0)$.
- c. $f(0) = \log_2 5 - 3 = \frac{\ln 5}{\ln 2} - 3 \approx 2.322 - 3 \approx -0.678$.
- d. Three convenient points: $(-4, -3)$, $(-1, -1)$, $(3, 0)$. The curve rises slowly and hugs the asymptote on the left.



13 Using a graph and logs



- a. The curve meets the line a little before $x = 3$ (since $3^3 = 27 > 20$) and after $x = 2$ (since $3^2 = 9 < 20$): about 2.7.

b. $3^x = 20$ gives $x = \log_3 20 = \frac{\ln 20}{\ln 3} \approx 2.727$, which agrees with the graph.

14 Growing bacteria



- a. $500 \cdot 2^{t/3} = 8000$ gives $2^{t/3} = 16 = 2^4$, so $t/3 = 4$ and $t = 12$ hours.

b. $2^{t/3} = 20$, so $\frac{t}{3} = \log_2 20 = \frac{\ln 20}{\ln 2} \approx 4.322$ and $t \approx 12.97$, about 13.0 hours.

15 A product of logs

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Condense: $\log_2(x(x-6)) = 4$, so $x^2 - 6x = 16$, i.e. $x^2 - 6x - 16 = 0$, i.e. $(x-8)(x+2) = 0$.

Candidates: $x = 8$ and $x = -2$. For $x = -2$, $\log_2 x$ and $\log_2(x-6)$ would involve negative numbers, which is impossible, so -2 is extraneous. Check $x = 8$: $\log_2 8 + \log_2 2 = 3 + 1 = 4$. The solution is $x = 8$.

16 A quotient of logs

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Condense: $\log \frac{x+3}{x-1} = 1$, so $\frac{x+3}{x-1} = 10$.

Then $x+3 = 10x-10$, so $13 = 9x$ and $x = \frac{13}{9} \approx 1.444$.

Check: $x > 1$, so both $x+3$ and $x-1$ are positive. The solution is $x = \frac{13}{9}$.

17 Saving for a goal

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a. $(1.036)^t = 1.5$, so $t = \frac{\ln 1.5}{\ln 1.036} \approx \frac{0.4055}{0.03537} \approx 11.46$ years.

b. $e^{0.036t} = 1.5$, so $t = \frac{\ln 1.5}{0.036} \approx 11.26$ years.

Continuous compounding is a little faster, by about 0.2 year (roughly 2 months).

18 Radioactive decay

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a. $(\frac{1}{2})^{t/12} = \frac{1}{16} = (\frac{1}{2})^4$, so $t = 48$ days.

b. $(\frac{1}{2})^{t/12} = \frac{1}{4}$, so $t/12 = 2$ and $t = 24$ days.

c. $(\frac{1}{2})^{t/12} = \frac{3}{8}$, so $\frac{t}{12} = \frac{\ln(3/8)}{\ln(1/2)} \approx 1.415$ and $t \approx 17.0$ days.

19 A hidden quadratic



Let $u = e^x$. The equation becomes $u^2 - 5u + 6 = 0$, i.e. $(u - 2)(u - 3) = 0$, so $u = 2$ or $u = 3$.

Then $e^x = 2$ gives $x = \ln 2 \approx 0.693$, and $e^x = 3$ gives $x = \ln 3 \approx 1.099$. Both are valid because e^x is always positive.

20 Finding an inverse



a. Write $y = 2 \cdot 3^x + 1$ and swap: $x = 2 \cdot 3^y + 1$. Then $3^y = \frac{x-1}{2}$, so

$$f^{-1}(x) = \log_3 \frac{x-1}{2}.$$

b. We need $\frac{x-1}{2} > 0$: the domain is $x > 1$ (the range of f).

c. $f^{-1}(55) = \log_3 27 = 3$. Check: $f(3) = 2 \cdot 27 + 1 = 55$.

21 Reciprocal logarithms



a. With natural logs: $\log_a b \cdot \log_b a = \frac{\ln b}{\ln a} \cdot \frac{\ln a}{\ln b} = 1$.

b. $\log_2 9 = 2 \log_2 3$ and $\log_3 8 = 3 \log_3 2$. Since $\log_2 3 \cdot \log_3 2 = 1$, the product is $2 \cdot 3 \cdot 1 = 6$.



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