

Linear Systems and Matrices: math test solutions, College – download the PDF

Test solutions with the detailed point scale. Add up your points and spot what to review.

Suggested time: 45 minutes. Out of 20 points. Calculator allowed only when the problem says so.

1 Gaussian elimination

/ 4 pts

Augmented matrix $\left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 1 & -1 & 2 & 5 \\ 3 & 2 & 1 & 10 \end{array} \right)$ (1 pt).

Swap $R_1 \leftrightarrow R_2$. Then $R_2 \leftarrow R_2 - 2R_1$ gives $(0, 3, -5 \mid -9)$ and $R_3 \leftarrow R_3 - 3R_1$ gives $(0, 5, -5 \mid -5)$ (1 pt).

Divide R_3 by 5: $(0, 1, -1 \mid -1)$. Swap $R_2 \leftrightarrow R_3$, then $R_3 \leftarrow R_3 - 3R_2$ gives $(0, 0, -2 \mid -6)$ (1 pt).

So $z = 3$, $y = z - 1 = 2$, $x = 5 + y - 2z = 1$. Solution $(1, 2, 3)$ (1 pt).

2 Free variable

/ 3 pts

$R_2 \leftarrow R_2 - 2R_1$ gives $(0, 0, 1 \mid 2)$, and $R_3 \leftarrow R_3 - R_1$ gives $(0, 0, -1 \mid -2)$ (1 pt).

Adding these rows leaves $0 = 0$, so the system is consistent, with $z = 2$ (1 pt).

Let $y = t$. From the first equation, $x = 3 - t - 4 = -1 - t$. Solution set: $(-1 - t, t, 2)$ for every real t (1 pt).

3 Matrix operations

/ 3 pts

$$2A - B = \begin{pmatrix} 2 & -4 \\ 5 & 2 \end{pmatrix} \text{ (1 pt).}$$

$$AB = \begin{pmatrix} 0+1 & 2-4 \\ 0-3 & 4+12 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -3 & 16 \end{pmatrix} \text{ (1 pt).}$$

$$BA = \begin{pmatrix} 0+4 & 0+6 \\ -1+8 & 1+12 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 7 & 13 \end{pmatrix} \text{ (1 pt).}$$

4 Inverse and solution

/ 4 pts

(a) $\det A = 9 - 8 = 1 \neq 0$, so A is invertible (1 pt).

$$(b) A^{-1} = \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix} \text{ (1 pt).}$$

$$(c) \begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \begin{pmatrix} 7 \\ 10 \end{pmatrix} = \begin{pmatrix} 21 - 20 \\ -28 + 30 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ (1 pt). Check: } 3 + 4 = 7 \text{ and } 4 + 6 = 10 \text{ (1 pt).}$$

5 A 3 by 3 determinant

/ 3 pts

Expand along the first row: $1 \cdot (1 \cdot 0 - (-1) \cdot 4) - 0 + 2 \cdot (3 \cdot 4 - 1 \cdot 2) = 4 + 20 = 24$ (1 pt).

$$\det A^T = 24 \text{ and } \det(2A) = 2^3 \cdot 24 = 192 \text{ (1 pt).}$$

Since $\det A \neq 0$, A is invertible (1 pt).

6 LU factorization

/ 3 pts

$R_2 \leftarrow R_2 - 2R_1$ gives $(0, 1, 3)$; R_3 already has a zero in column 1 (multiplier 0);

$R_3 \leftarrow R_3 - 3R_2$ gives $(0, 0, -4)$ (1 pt).

$$U = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & -4 \end{pmatrix} \text{ and } L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix} \text{ (1 pt).}$$

Forward: $y_1 = 2$, $y_2 = 1 - 4 = -3$, $y_3 = -5 + 9 = 4$. Back: $z = -1$, $y = -3 + 3 = 0$, $x = 2$. Solution $(2, 0, -1)$ (1 pt).



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