

Linear Systems and Matrices: practice solutions, College – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Two lines, one point



Adding the two equations eliminates y : $3x = 9$, so $x = 3$. From $x - y = 2$ we get $y = 1$.

(1) Check: $2 \cdot 3 + 1 = 7$ and $3 - 1 = 2$. (2) The graph confirms it: the lines cross at $Q(3, 1)$.

Answer: $(x, y) = (3, 1)$, the unique solution.

2 Writing the augmented matrix



Missing unknowns get coefficient 0: $\left(\begin{array}{ccc|c} 3 & -1 & 2 & 5 \\ 1 & 0 & 4 & -1 \\ 0 & 2 & -1 & 7 \end{array} \right)$.

After $R_1 \leftrightarrow R_2$: rows $(1, 0, 4 \mid -1)$, $(3, -1, 2 \mid 5)$, $(0, 2, -1 \mid 7)$. After $R_2 \leftarrow R_2 - 3R_1$: the second row is $(0, -1, -10 \mid 8)$ because $5 - 3(-1) = 8$ and $2 - 12 = -10$.

$$\left(\begin{array}{ccc|c} 1 & 0 & 4 & -1 \\ 0 & -1 & -10 & 8 \\ 0 & 2 & -1 & 7 \end{array} \right)$$

3 Size and entries



- a. A has 3 rows and 4 columns, so it is 3×4 .
- b. a_{23} is in row 2, column 3: 5 . $a_{14} = 7$ and $a_{32} = 9$.
- c. Transposing swaps rows and columns, so A^T is 4×3 . The entry $(A^T)_{32}$ equals $a_{23} = 5$.

4 Adding and scaling matrices



Work entry by entry. $A + B = \begin{pmatrix} 1+4 & -2+1 \\ 3-1 & 0+5 \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 2 & 5 \end{pmatrix}$.

$$3A = \begin{pmatrix} 3 & -6 \\ 9 & 0 \end{pmatrix} \text{ and } 2B = \begin{pmatrix} 8 & 2 \\ -2 & 10 \end{pmatrix}, \text{ so } 3A - 2B = \begin{pmatrix} -5 & -8 \\ 11 & -10 \end{pmatrix}.$$

5 Two small determinants



First: $5 \cdot 4 - 2 \cdot 3 = 20 - 6 = 14$. Second: $6 \cdot 2 - (-3)(-4) = 12 - 12 = 0$.

Since a matrix is invertible exactly when its determinant is nonzero, only the first matrix is invertible; the second is singular (its second row is $-\frac{2}{3}$ times its first row, so the rows are proportional).

6 True or false?



- a. **False.** The system $x + y + z = 1$, $x + y + z = 2$ has three unknowns and two equations, yet it is inconsistent. More unknowns only guarantees infinitely many solutions *if* the system is consistent.
- b. **True.** Setting every unknown to 0 satisfies every equation, so the all-zero list is always a solution.
- c. **False.** If A is 2×3 and B is 3×4 , then AB is defined, but BA would need the 4 columns of B to match the 2 rows of A , which they do not.

7 Inverse of a 2 by 2 matrix



$$\det A = 4 \cdot 2 - 7 \cdot 1 = 1, \text{ so } A^{-1} = \frac{1}{1} \begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix}.$$

$$\text{Then } \begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 - 7 \\ -3 + 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

Check: $4(-1) + 7(1) = 3$ and $-1 + 2(1) = 1$. Answer: $(x, y) = (-1, 1)$.

8 Elimination with three unknowns



$$\text{Augmented matrix: } \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 5 & 3 & 6 \\ -1 & 1 & 4 & -5 \end{array} \right).$$

$R_2 \leftarrow R_2 - 2R_1$ gives $(0, 1, 1 \mid 0)$ and $R_3 \leftarrow R_3 + R_1$ gives $(0, 3, 5 \mid -2)$. Then $R_3 \leftarrow R_3 - 3R_2$ gives $(0, 0, 2 \mid -2)$.

Back-substitute: $z = -1$; $y + z = 0$ gives $y = 1$; $x + 2 - 1 = 3$ gives $x = 2$.

Answer: $(2, 1, -1)$. Check in the third equation: $-2 + 1 - 4 = -5$.

9 An inconsistent system



Apply $R_2 \leftarrow R_2 - 2R_1$: the second row becomes $(0, 0, 0 \mid 3)$, which says $0 = 3$. That is impossible, so the system is inconsistent.

Geometrically, the left side of the second equation is exactly twice the left side of the first, but the right side is not twice as large: the first two planes are **parallel and distinct**, so they never meet, and no point can lie on all three planes.

10 Describing a solution set



Pivots are in columns 1 and 2, so x_1, x_2 are basic and x_3, x_4 are free. Let $x_3 = s$ and $x_4 = t$.

Row 1: $x_1 + 2s = 5$, so $x_1 = 5 - 2s$. Row 2: $x_2 - s + 3t = 1$, so $x_2 = 1 + s - 3t$.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ -3 \\ 0 \\ 1 \end{pmatrix}, \text{ a plane of solutions in four-dimensional space.}$$

11 AB is not BA



$$AB = \begin{pmatrix} 2 \cdot 1 + (-1) \cdot 5 & 2 \cdot 4 + (-1)(-2) \\ 0 \cdot 1 + 3 \cdot 5 & 0 \cdot 4 + 3(-2) \end{pmatrix} = \begin{pmatrix} -3 & 10 \\ 15 & -6 \end{pmatrix}.$$

$$BA = \begin{pmatrix} 1 \cdot 2 + 4 \cdot 0 & 1(-1) + 4 \cdot 3 \\ 5 \cdot 2 + (-2) \cdot 0 & 5(-1) + (-2) \cdot 3 \end{pmatrix} = \begin{pmatrix} 2 & 11 \\ 10 & -11 \end{pmatrix}.$$

The matrices differ, so $AB \neq BA$: multiplication is not commutative.

12 Cofactor expansion



Signs are $+, -, +$. The first term is $1 \cdot ((-1)(-2) - 4 \cdot 5) = 1 \cdot (2 - 20) = -18$. The second is $-2 \cdot (3(-2) - 4 \cdot 2) = -2 \cdot (-14) = 28$. The third is 0.

$\det = -18 + 28 + 0 = 10$. Since $10 \neq 0$, the matrix is invertible.

13 Echelon forms



- Pivots are in columns 1 and 3, each equal to 1 with zeros elsewhere in its column, and the zero row is at the bottom: **RREF**.
- A staircase of pivots, but the 2 and 3 above pivots are nonzero: **REF but not RREF**.
- The pivot of row 2 (column 1) is to the left of the pivot of row 1 (column 2): **neither**.
- A staircase, but the first pivot is 2, not 1: **REF but not RREF**.

14 Trail mix



Let p, r, c be the pounds of peanuts, raisins, and chips. Then $p + r + c = 10$, $4p + 3r + 6c = 40$, and $p - 2c = 0$.

From the third, $p = 2c$. Substituting: $3c + r = 10$, so $r = 10 - 3c$; and $8c + 3(10 - 3c) + 6c = 40$, which simplifies to $5c + 30 = 40$, so $c = 2$. Then $p = 4$ and $r = 4$.

Check: $4 + 4 + 2 = 10$ lb and $16 + 12 + 12 = 40$ dollars.

Answer: 4 lb of peanuts (about 1.81 kg), 4 lb of raisins (about 1.81 kg), and 2 lb of chips (about 0.91 kg).

15 Determinant rules



- a. $\det(2A) = 2^3 \cdot 5 = 40$ (each of the 3 rows is doubled).
- b. $\det A^T = \det A = 5$.
- c. $\det A^{-1} = 1/5$.
- d. $\det(A^2) = (\det A)^2 = 25$.
- e. A row swap flips the sign: -5 .
- f. Adding a multiple of a row does not change the determinant: 5 .

16 Solve with an LU factorization



$$A = LU = \begin{pmatrix} 3 & 1 \\ 6 & 6 \end{pmatrix} \text{ (second row: } 2 \cdot (3, 1) + (0, 4) = (6, 6)\text{)}.$$

Forward substitution in $L\mathbf{y} = (5, 18)$: $y_1 = 5$, $y_2 = 18 - 2 \cdot 5 = 8$. Back substitution in $U\mathbf{x} = \mathbf{y}$: $4x_2 = 8$ gives $x_2 = 2$; $3x_1 + 2 = 5$ gives $x_1 = 1$.

Answer: $\mathbf{x} = (1, 2)$. Check: $3 + 2 = 5$ and $6 + 12 = 18$.

17 Inverse by row reduction

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Start with $\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$.

$R_2 \leftarrow R_2 - 3R_3$ gives $(0, 1, 0 \mid 0, 1, -3)$. Then $R_1 \leftarrow R_1 - 2R_2$ gives $(1, 0, 0 \mid 1, -2, 6)$.

$A^{-1} = \begin{pmatrix} 1 & -2 & 6 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$. Check the first row of AA^{-1} : $(1, -2 + 2, 6 - 6) = (1, 0, 0)$;

second row: $(0, 1, -3 + 3) = (0, 1, 0)$; third row: $(0, 0, 1)$. So $AA^{-1} = I$.

18 A parameter in the system

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Apply $R_2 \leftarrow R_2 - 2R_1$: the system becomes $x + 2y = 3$ and $(k - 4)y = 1$.

If $k \neq 4$, then $y = \frac{1}{k-4}$ and $x = 3 - \frac{2}{k-4}$: a **unique solution**.

If $k = 4$, the second equation reads $0 = 1$: **no solution**.

The system never has infinitely many solutions, because the right-hand side **1** never vanishes.

19 Solving a matrix equation

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$\det A = 1$, so $A^{-1} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$. Multiplying $AX = B$ on the **left** by A^{-1} gives $X = A^{-1}B$.

$$X = \begin{pmatrix} 3 \cdot 4 - 11 & 3 \cdot 2 - 5 \\ -5 \cdot 4 + 2 \cdot 11 & -5 \cdot 2 + 2 \cdot 5 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}.$$

$$\text{Check: } AX = \begin{pmatrix} 2+2 & 2+0 \\ 5+6 & 5+0 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 11 & 5 \end{pmatrix} = B.$$

20 Inverse of a product

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(a) Multiply: $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AIA^{-1} = AA^{-1} = I$, using associativity. By definition of the inverse, $B^{-1}A^{-1}$ is $(AB)^{-1}$. The order reverses because B must be undone first.

(b) $AB = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$ with determinant 1, so $(AB)^{-1} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$. Also $A^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$, and $B^{-1}A^{-1} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$. The two results agree.

21 Build and use an LU factorization

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$R_2 \leftarrow R_2 - 3R_1$ gives $(0, 1, 2)$ (multiplier 3). $R_3 \leftarrow R_3 - 2R_1$ gives $(0, 3, 8)$ (multiplier 2). $R_3 \leftarrow R_3 - 3R_2$ gives $(0, 0, 2)$ (multiplier 3).

$$U = \begin{pmatrix} 2 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}, L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix}.$$

Forward: $y_1 = 2$, $y_2 = 9 - 6 = 3$, $y_3 = 17 - 4 - 9 = 4$. Back: $z = 2$, $y = 3 - 4 = -1$, $2x + 4(-1) + 2(2) = 2$ so $x = 1$.

Answer: $\mathbf{x} = (1, -1, 2)$.

22 When is a matrix singular?

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A square matrix is singular exactly when its determinant is 0. Expand along the first row:

$$\det A = x(x \cdot x - 1 \cdot 1) - 1(1 \cdot x - 1 \cdot 0) + 0 = x^3 - x - x = x^3 - 2x.$$

$$x^3 - 2x = x(x^2 - 2) = 0 \text{ gives } x = 0 \text{ or } x = \pm\sqrt{2}.$$

Answer: A is singular exactly for $x \in \{-\sqrt{2}, 0, \sqrt{2}\}$.



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