

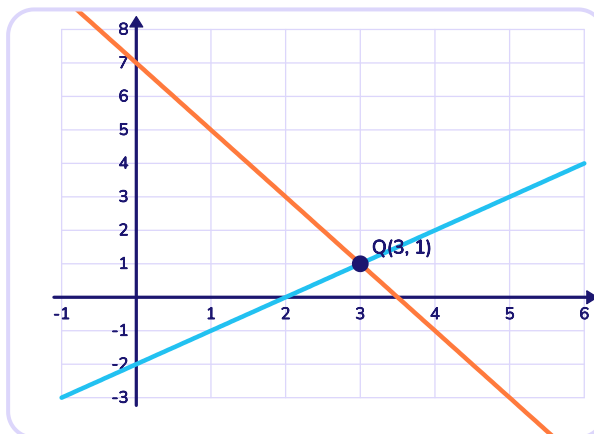
Linear Systems and Matrices: math practice, College – download the PDF

22 problems graded by difficulty (★ easy, ★★ medium, ★★★ hard). Try first, without looking at the solutions!

1 Two lines, one point

★ ★ ★

Solve the system $2x + y = 7$ and $x - y = 2$ by adding the equations. Then check the answer against the graph.



2 Writing the augmented matrix

★ ★ ★

Write the augmented matrix of $3x - y + 2z = 5$, $x + 4z = -1$, $2y - z = 7$. Then swap rows 1 and 2 and apply $R_2 \leftarrow R_2 - 3R_1$. Write the new matrix.

3 Size and entries



Let $A = \begin{pmatrix} 4 & -2 & 0 & 7 \\ 1 & 3 & 5 & -6 \\ 0 & 9 & 2 & 8 \end{pmatrix}$. (a) What is the size of A ? (b) Find a_{23} , a_{14} , and a_{32} . (c)

What is the size of A^T , and which entry of A^T equals a_{23} ?

4 Adding and scaling matrices



Let $A = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 1 \\ -1 & 5 \end{pmatrix}$. Compute $A + B$ and $3A - 2B$.

5 Two small determinants



Compute $\det \begin{pmatrix} 5 & 2 \\ 3 & 4 \end{pmatrix}$ and $\det \begin{pmatrix} 6 & -3 \\ -4 & 2 \end{pmatrix}$. Which matrix is invertible?

6 True or false?



Decide whether each statement is true or false, and justify your answer. (a) A system with more unknowns than equations always has infinitely many solutions. (b) A system where every right-hand side is 0 always has at least one solution. (c) If AB is defined, then BA is defined.

7 Inverse of a 2 by 2 matrix



Find the inverse of $A = \begin{pmatrix} 4 & 7 \\ 1 & 2 \end{pmatrix}$, then use it to solve $4x + 7y = 3$, $x + 2y = 1$.

8 Elimination with three unknowns



Solve by Gaussian elimination: $x + 2y + z = 3$, $2x + 5y + 3z = 6$, $-x + y + 4z = -5$.

9 An inconsistent system

★★★

Show that $x + 2y - z = 1$, $2x + 4y - 2z = 5$, $x - y = 0$ has no solution, and explain why geometrically.

10 Describing a solution set

★★★

The reduced echelon form of a system in unknowns x_1, x_2, x_3, x_4 is $\left(\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 5 \\ 0 & 1 & -1 & 3 & 1 \end{array}\right)$. Identify the basic and free variables, and write the general solution in vector form.

11 AB is not BA

★★★

Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 4 \\ 5 & -2 \end{pmatrix}$. Compute AB and BA . Are they equal?

12 Cofactor expansion

★★★

Compute $\det \begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \\ 2 & 5 & -2 \end{pmatrix}$ by expanding along the first row.

13 Echelon forms

★★★

For each matrix, say whether it is in row echelon form, reduced row echelon form, or

neither. (a) $\begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

14 Trail mix

A shop mixes peanuts, raisins, and chocolate chips into 10 pounds of trail mix. Peanuts cost 4 dollars per pound, raisins 3 dollars per pound, and chips 6 dollars per pound. The mix costs 40 dollars in all, and it contains twice as many pounds of peanuts as of chips. How much of each (in pounds, and in kilograms, using $1 \text{ lb} = 0.4536 \text{ kg}$) is used?

15 Determinant rules

A 3×3 matrix A has $\det A = 5$. Find (a) $\det(2A)$, (b) $\det(A^T)$, (c) $\det(A^{-1})$, (d) $\det(A^2)$, (e) the determinant after swapping two rows of A , (f) the determinant after replacing R_2 by $R_2 + 3R_1$.

16 Solve with an LU factorization

Let $A = LU$ with $L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ and $U = \begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix}$. Compute A , then solve $A\mathbf{x} = (5, 18)$ using L and U .

17 Inverse by row reduction

Use $[A \mid I]$ to find A^{-1} for $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$, and verify the answer by multiplying.

18 A parameter in the system

For which values of k does the system $x + 2y = 3$, $2x + ky = 7$ have a unique solution, no solution, or infinitely many solutions?

19 Solving a matrix equation

★★★

Let $A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 2 \\ 11 & 5 \end{pmatrix}$. Find the matrix X with $AX = B$, and verify.

20 Inverse of a product

★★★

(a) Prove that if A and B are invertible $n \times n$ matrices, then $(AB)^{-1} = B^{-1}A^{-1}$. (b)

Check this for $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$.

21 Build and use an LU factorization

★★★

Find L and U for $A = \begin{pmatrix} 2 & 4 & 2 \\ 6 & 13 & 8 \\ 4 & 11 & 12 \end{pmatrix}$ using no row swaps, then solve $A\mathbf{x} = (2, 9, 17)$.

22 When is a matrix singular?

★★★

Find all real numbers x for which $A = \begin{pmatrix} x & 1 & 0 \\ 1 & x & 1 \\ 0 & 1 & x \end{pmatrix}$ is singular.

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